Math 111-002
Assignment # 10

Please remember that the assignment consists of only a sample of the kind of questions you are supposed to be able to do. It is **not** a safe practice to just do the assignment, and that is why there is a list of "suggested practice problems" in the course web page.

1. Suppose that q is a real number with $|q| < 1$.

   (a) Use a trick similar to the one we used in class to find the sum of the geometric series to show that

   $$\sum_{k=M}^{N} q^k = \frac{q^M - q^{N+1}}{1-q}.$$

   (b) Prove that

   $$\sum_{k=M}^{\infty} q^k = \frac{q^M}{1-q}.$$

2. The decimal notation we use to write numbers is based on powers of 10. For example, when we write 543.25, we really mean $5 \times 10^2 + 4 \times 10^1 + 3 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$. In particular, when we write $0.9999999\ldots$ we mean $9 \sum_{k=1}^{\infty} \frac{1}{10^k}$. Use this fact to prove that

   (a) $0.33333\ldots = 1/3$;

   (b) $0.9999999\ldots = 1$.

3. Use a geometric series to write $0.87\overline{43}$ as a fraction.

4. Determine if the series is convergent or divergent. If it converges, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-7)^{n-1}}{6^{n-1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{7^{n-1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n^2+1}$