Math 111-002
Assignment # 12

Please remember that the assignment consists of only a sample of the kind of questions you are supposed to be able to do. It is **not** a safe practice to just do the assignment, and that is why there is a list of "suggested practice problems" in the course web page.

1. Look at the trick we used to find power series representations of $\ln(1+x)$ and $\arctan x$. Use an analog trick – but with differentiation instead of integration – to find a power series representation of
$$f(x) = \frac{1}{(1-x)^2}, \quad |x| < 1.$$

2. Find power series representations for

   (a) $\dfrac{x^3}{(1-x)^2}$

   (b) $\dfrac{1}{(1-x)^3}$

   (c) $\left(\dfrac{x}{2-x}\right)^3$.

3. Find the radius and interval of convergence, and an explicit formula for

   (a) $\displaystyle\sum_{n=1}^{\infty} nx^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} n^2(x-3)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$.

4. Use a power series to find $\displaystyle\int_0^{1/2} \arctan\frac{x}{2}\,dx$ with three good decimals.

5. Find the Taylor polynomial of each function at the given point a.

   (a) $f(x) = \cos x$, $a = \pi/2$;

   (b) $f(x) = e^{-x}$, $a = 0$.

6. Use the Taylor polynomials found in class to calculate the following:

   (a) The number e, with 6 good decimals.

   (b) The number $1/e$ with 6 good decimals.

   (c) $\arctan\frac{1}{2}$ with 7 good decimals.