

1. Consider the solid obtained by rotating the region bounded by $y = \sqrt[3]{x}$, $x = 8$, around the x -axis. Find the volume:

- (a) By disks/slices;

Answer.

$$V = \pi \int_0^8 x^{2/3} dx = \pi \frac{8^{5/3}}{5/3} = \pi \frac{3 \times 2^5}{5} = \frac{96\pi}{5}.$$

- (b) By cylindrical shells.

Answer. We need to integrate on the y -axis. When $x = 8$, $y = \sqrt[3]{8} = 2$. Since $y = x^{1/3}$, we get $x = y^3$. We need to subtract from the full rectangle. Thus

$$V = 2\pi \int_0^2 y(8 - y^3) dy = 2\pi \int_0^2 (8y - y^4) dy = 2\pi \left(4 \times 2^2 - \frac{2^5}{5} \right) = \frac{96\pi}{5}.$$