Math 111-002 201710
Quiz # 2 - Answers

1. Find $(f^{-1})'(6)$, where $f(x) = x + \sqrt{x}$.

   Answer. Testing on the numbers whose square roots are easy, we see that $f(1) = 2$ (not useful to us) and $f(4) = 4 + \sqrt{4} = 4 + 2 = 6$. So $f^{-1}(6) = 4$. We have $f'(x) = 1 + \frac{1}{2\sqrt{x}}$. Using the formula for the derivative of the inverse,

$$(f^{-1})'(6) = \frac{1}{f'(f^{-1}(6))} = \frac{1}{f'(4)} = \frac{1}{1 + \frac{1}{2\sqrt{4}}} = \frac{1}{1 + \frac{1}{4}} = \frac{4}{5}.$$

2. Evaluate $\displaystyle\int_0^1 \frac{x}{1 + x^4}\, dx$.

   Answer. Using the substitution $u = x^2$, we have $du = 2x\, dx$. In the limits, $u = 0$ when $x = 0$ and $u = 1$ when $x = 1$. Then

$$\int_0^1 \frac{x}{1 + x^4}\, dx = \frac{1}{2} \int_0^1 \frac{1}{1 + u^2}\, du = \frac{1}{2}\ \arctan u \Big|_0^1 = \frac{1}{2} \frac{\pi}{4} = \frac{\pi}{8}.$$