Math 111-002 201710
Quiz # 3 - Answers

1. Evaluate $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}}\,dx$.

Answer. We make the substitution $x = 2\sin t$. Then $dx = 2\cos t\,dt$, and $\sqrt{4-4\sin^2 t} = 2\cos t$. Then

$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{8\sin^3 t\, 2\cos t}{2\cos t}\,dt = 8\int sin^3 t\,dt = 8\int \sin t\,\sin^2 t\,dt = 8\int \sin t\,(1-\cos^2 t)\,dt$$

$$\textit{(Substitution: } v = \cos t,\ dv = -\sin t\,dt\textit{)}$$

$$= 8\int \sin t\,dt - 8\int \sin t\,\cos^2 t\,dt = -8\cos t + 8\int v^2\,dv$$

$$= -8\cos t + \frac{8v^3}{3} = -8\cos t + \frac{8\cos^3 t}{3}$$

$$\textit{(Using } \cos t = \frac{1}{2}\sqrt{4-x^2}\textit{)}$$

$$= -4\sqrt{4-x^2} + \frac{8}{3}\frac{1}{8}(4-x^2)^{3/2} = -\frac{1}{3}\sqrt{4-x^2}\,(12-(4-x^2)) =$$

$$-\frac{1}{3}\sqrt{4-x^2}\,(x^2+8).$$

2. Evaluate $\displaystyle\int_0^1 \frac{1}{(x+1)(x+2)}\,dx$.

Answer. We use partial fractions:

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{(A+B)x + 2A + B}{(x+1)(x+2)}.$$

Comparing coefficients we get $A+B=0$, $2A+B=1$, so $A=1$, $B=-1$. Then

$$\int_0^1 \frac{1}{(x+1)(x+2)}\,dx = \int_0^1 \frac{1}{x+1}\,dx - \int_0^1 \frac{1}{x+2}\,dx$$

$$= \ln(x+1)\Big|_0^1 - \ln(x+2)\Big|_0^1$$

$$= \ln 2 - \ln 3 + \ln 2 = \ln\frac{4}{3}.$$