Math 111-002 201710
Quiz # 4 - Answers

1. Decide whether each integral is convergent or divergent. Evaluate those that are convergent.

   (a) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx.$

   Answer. The integral is improper on both limits, so we need to split it. Most commonly this is done at $x = 0$, since it simplifies our calculations. We have, since $\arctan 0 = 0$,

$$\int_0^{\infty} \frac{1}{1+x^2}\,dx = \lim_{M\to\infty} \int_0^{M} \frac{1}{1+x^2}\,dx = \lim_{M\to\infty} \arctan x\Big|_0^M = \lim_{M\to\infty} \arctan M = \frac{\pi}{2}.$$

   Now since $f(x) = \frac{1}{1+x^2}$ is an even function, we have $\in_{-M}^0 \frac{1}{1+x^2}\,dx = \int_0^M \frac{1}{1+x^2}\,dx$, so

$$\int_{-\infty}^{0} \frac{1}{1+x^2}\,dx = \frac{\pi}{2},$$

   and then

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_0^{\infty} \frac{1}{1+x^2}\,dx = \frac{\pi}{2} + \frac{\pi}{2} = \pi.$$

   (b) $\displaystyle\int_2^{\infty} \frac{5+e^{-x}}{x}\,dx$

   Answer. We have, since the exponential is always positive,

$$\frac{5+e^{-x}}{x} \geq \frac{5}{x}.$$

   Then, by comparison,

$$\int_2^{\infty} \frac{5+e^{-x}}{x}\,dx \geq \int_2^{\infty} \frac{5}{x}\,dx = \infty.$$