Math 111-002 201710
Quiz # 5 – Answers

1. Use a geometric series to write $0.4\overline{93} = 0.493939393\ldots$ as a fraction.

   Answer. We have

$$0.4\overline{93} = \frac{4}{10} + \frac{1}{10}\sum_{n=1}^{\infty}\frac{93}{100^n} = \frac{4}{10} + \frac{93}{10}\sum_{n=1}^{\infty}\left(\frac{1}{100}\right)^n$$

$$= \frac{4}{10} + \frac{93}{10}\frac{\frac{1}{100}}{1-\frac{1}{100}} = \frac{4}{10} + \frac{93}{10}\frac{1}{99}$$

$$= \frac{4\times 99 + 93}{990} = \frac{396+93}{990}$$

$$= \frac{489}{990}.$$

2. Determine if the series is convergent.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{3}{\sqrt{n^2+4n}}$

   Answer. Since $\sqrt{n^2+4n}$ is roughly n for big n, we can compare with the harmonic series. We have

$$\lim_{n\to\infty}\frac{\frac{1}{n}}{\frac{3}{\sqrt{n^2+4n}}} = \lim_{n\to\infty}\frac{\sqrt{n^2+4n}}{n} = \lim_{n\to\infty}\sqrt{1+4/n} = 1.$$

   So, by the Limit Comparison Theorem, our series behaves the same as the harmonic series, so it diverges.

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n^2+4n}}$

   Answer. This is an alternating series and the terms go to zero as $n\to\infty$. To apply Leibnitz's criterion, we should check whether the terms decrease. We have, if $f(x) = 1/\sqrt{x^2+4x}$,

$$f'(x) = -\frac{1}{2}\frac{x+2}{(x^2+4x)^{3/2}}.$$

   When $x \geq 1$, the quotient above is positive, and so the whole expression is negative; thus $f'(x) < 0$, and then the sequence $\{1/\sqrt{n^2+4n}\}$ is decreasing. By Leibnitz's criterion, the series converges.