Mathematics 111 (Calculus II) Laboratory Manual

Department of Mathematics & Statistics University of Regina

2nd edition

prepared by Patrick Maidorn, Fotini Labropulu, and Robert Petry

University of Regina Department of Mathematics and Statistics

Contents

Introduction

"*One does not learn how to swim by reading a book about swimming,*" as surely everyone agrees. The same is true of mathematics. One does not learn mathematics by only reading a textbook and listening to lectures. Rather, one learns mathematics by doing mathematics.

This Laboratory Manual is a small set of problems that are representative of the types of problems that students of Mathematics 111 (Calculus II) at the University of Regina are expected to be able to solve on quizzes, midterm exams, and final exams. In the weekly lab of your section of Math 111 you will work on selected problems from this manual under the guidance of the laboratory instructor, thereby giving you the opportunity to do mathematics with a coach close at hand. These problems are not homework and your work on these problems will not be graded. However, by working on these problems during the lab periods, and outside the lab periods if you wish, you will gain useful experience in working with the central ideas of elementary calculus.

The material in the Lab Manual does not replace the textbook. There are no explanations or short reviews of the topics under study. Thus, you should refer to the relevant sections of your textbook and your class notes when using the Lab Manual. These problems are not sufficient practice to master calculus, and so you should solidify your understanding of the material by working through problems given to you by your professor or that you yourself find in the textbook.

To succeed in calculus it is imperative that you attend the lectures and labs, read the relevant sections of the textbook carefully, and work on the problems in the textbook and laboratory manual. Through practice you will learn, and by learning you will succeed in achieving your academic goals. We wish you good luck in your studies of calculus.

Module 1

Inverse Functions

1.1 Inverse Functions

1. Determine whether each of the following functions is invertible on its domain.

   (a) $f(x) = 1 + x^4$

   (b) $f(x) = \sin x + \cos x$

   (c) $f(x) = \dfrac{x}{x-1}$

   (d) $f(x) = \cos x$ on $x \in \left[0, \dfrac{3\pi}{2}\right]$

   (e) $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0 \end{cases}$

Answers: Page 22

2. In each case, find a formula for the inverse function $f^{-1}(x)$.

   (a) $f(x) = x^3 - 1$

   (b) $f(x) = x^2 - 4x$ on $x \geq 2$

   (c) $f(x) = \dfrac{1-x}{2+x}$

3. In each case, check whether $f(x)$ is a one to one differentiable function, and if it is, find $\left(f^{-1}\right)'(a)$.

   (a) $f(x) = x^5 + 3x^3 + 4$, $a = 8$

   (b) $f(x) = 1 - \sin x$, $a = \dfrac{1}{2}$

   (c) $f(x) = x^3 - x$, $a = 2$

1.2 Exponential and Logarithmic Functions

1. Write each expression as a single exponential.

   (a) $\left(2^{3+x}\right) \cdot \dfrac{3^x}{2^3}$

   (b) $\dfrac{e^{\frac{5x}{3}}}{\sqrt[3]{e^x}}$

Answers: Page 22

2. Write as a single logarithm.

   (a) $4\ln(x) - \dfrac{1}{2}\ln(3x) + 1$

   (b) $4\log(x+1) - 2\log(x) + 2$

3. Solve each equation for x.

   (a) $3^{x-1} = 81$

   (b) $\left(\frac{1}{2}\right)^{x+1} = \left(\frac{1}{4}\right)^{x+2}$

   (c) $4e^{3x} = 16$

   (d) $2 = 10^{4x-2}$

   (e) $\ln(\ln(x)) = 1$

4. Assume the world's population doubles every 53 years.

   (a) Find its annual growth rate k in $N(t) = N_0 e^{kt}$.

   (b) In 1998, Earth's population was 6 billion. Use the model in (a) to predict the population in 2020.

   (c) In what year will Earth's population reach 10 billion, according to this model?

5. Radioactive carbon-14 has a half-life of 5730 years. How long will it take for an object to lose 80% of its original C-14 content?

1.3 Calculus of Exponential and Logarithmic Functions

1. Find the indicated derivatives.

Answers: Page 22

   (a) $f(x) = e^{x^2} + 3e^{4x}$, $f'(x)$

   (b) $f(x) = e^{\sin x}$, $f''(x)$

   (c) $f(x) = \dfrac{e^{\sqrt{x}}}{x}$, $f'(x)$

   (d) $f(x) = \ln(1 + x^2 + x)$, $f'(0)$

   (e) $f(x) = \ln(\cos x)$, $f''(x)$

   (f) $f(x) = \ln\left(\dfrac{x^2}{x^2+1}\right)$, $f'(x)$

2. Integrate the following:

   (a) $\displaystyle\int e^{4x} + \frac{4}{x}\,dx$

   (b) $\displaystyle\int_0^1 \frac{e^{2x} - 1}{e^x}\,dx$

   (c) $\displaystyle\int \frac{dx}{x-1}$

   (d) $\displaystyle\int x^2 e^{x^3+3}\,dx$

   (e) $\displaystyle\int \frac{3x}{x^2+1}\,dx$

   (f) $\displaystyle\int_0^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

3. Use logarithmic differentiation to find $f'(x)$.

   (a) $f(x) = x^x$

   (b) $f(x) = x^{\sin x}$

   (c) $f(x) = \dfrac{(2x+3)^{\frac{1}{2}}(x-7)^5}{(x+1)^{\frac{1}{6}}}$

4. Find the equation of the tangent line to $f(x) = e^{-x^2}$ at the point $x = 1$.

5. Find the area between $y = e^x$ and $y = e^{-x}$ between $x = -1$ and $x = 1$.

1.4 Inverse Trigonometric Functions

1. Find the exact value of each expression.

   (a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

   (b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

   (c) $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

   (d) $\cos^{-1}(\cos 3\pi)$

Answers: Page 23

2. Differentiate the following:

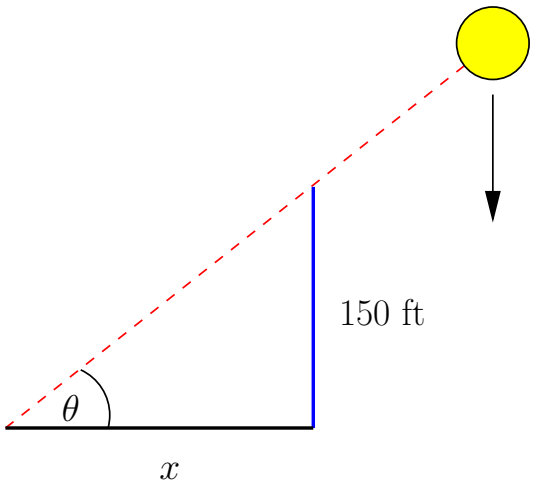
   (a) $f(x) = 3\cos^{-1} x + 5\tan^{-1} x$

   (b) $f(x) = \tan^{-1}\sqrt{3x}$

   (c) $f(x) = \sin^{-1}\sqrt{1-x^4}$

   (d) $f(x) = \cos\left(\sin^{-1} x\right)$

3. Find the equation of the tangent line to the graph of $f(x) = \sec^{-1} 2x$ at the point $(x, y) = \left(1, \frac{\pi}{3}\right)$.

4. 

   As the sun descends, the shadow cast by a 150 ft tall wall lengthens.

   (a) Express the angle θ as a function of the shadow's length x.

   (b) Find $\frac{d\theta}{dx}$ when the shadow's length is 200 ft.

5. Integrate the following:

   (a) $\int_{\frac{\sqrt{3}}{3}}^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

   (b) $\int \frac{x}{\sqrt{1-x^4}}\,dx$

   (c) $\int \frac{\tan^{-1} x}{x^2+1}\,dx$

1.5 L'Hopital's Rule

1-10: In each case, find the indicated limit. Note: not all limits allow the use of L'Hopital's Rule.

Answers: Page 23

1. $\lim_{x\to 0} \dfrac{\sin x^2}{x}$
2. $\lim_{x\to \infty} \dfrac{x^2+x+1}{4x^2+3}$
3. $\lim_{x\to 0} \dfrac{\ln(1+x)}{x}$
4. $\lim_{x\to 1} \dfrac{x-1}{x^2+1}$
5. $\lim_{x\to \infty} \dfrac{x^5+4x}{e^x}$
6. $\lim_{x\to 0} x\csc x$
7. $\lim_{x\to 0^+} x^{2x}$
8. $\lim_{x\to 0} (x+\cos x)^{\frac{1}{x}}$
9. $\lim_{x\to 0^+} \left(\dfrac{1}{\sin x} - \dfrac{1}{x}\right)$
10. $\lim_{x\to \infty} \left(\dfrac{1}{x} - \dfrac{1}{1-e^x}\right)$

Module 2

Techniques of Integration

2.1 Integration by Parts

1-10: Integrate the following:

Answers: Page 23

1. $\int x \sin 2x \, dx$
2. $\int 2xe^{-3x} \, dx$
3. $\int 3x^2 \cos x \, dx$
4. $\int_1^e x^2 \ln(x) \, dx$
5. $\int_0^{2\pi} (3x+5) \cos\left(\frac{x}{4}\right) \, dx$
6. $\int (\ln(x))^2 \, dx$
7. $\int x\sqrt{x+1} \, dx$
8. $\int \cos(2x)e^{3x} \, dx$
9. $\int ax^2 e^{bx} \, dx$
10. $\int x^5 \sin x \, dx$

2.2 Trigonometric Integrals

1-9: Integrate the following:

Answers: Page 24

1. $\int \sin^2 x \cos^5 x \, dx$
2. $\int \sin^5 x \cos^2 x \, dx$
3. $\int_0^{\pi} \sin^2 x \cos^2 x \, dx$
4. $\int \cos^4 x \, dx$
5. $\int \tan^3 x \sec^4 x \, dx$
6. $\int \sqrt{\tan x} \sec^4 x \, dx$
7. $\int \tan(2t) \sec^3(2t) \, dt$
8. $\int_0^{\frac{\pi}{2}} \cos^5 x \, dx$
9. $\int \tan^5 x \, dx$

2.3 Trigonometric Substitution

1-9: Integrate the following:

Answers: Page 24

1. $\int x^3\sqrt{1-x^2}\,dx$
2. $\int x^2\sqrt{4-x^2}\,dx$
3. $\int_{\frac{1}{5}}^{\frac{2}{5}} \frac{\sqrt{25x^2-1}}{x}\,dx$
4. $\int \frac{dx}{(4+x^2)^2}$
5. $\int \frac{x^3}{\sqrt{2-x^2}}$
6. $\int_0^{\frac{\sqrt{2}}{8}} \frac{dx}{(16-x^2)^{\frac{3}{2}}}$
7. $\int \frac{dx}{2x^2-12x+26}$
8. $\int_1^4 \frac{\sqrt{x^2+4x-5}}{x+2}\,dx$
9. $\int \left(x^2-6x+13\right)^{-\frac{1}{2}}\,dx$

2.4 Partial Fractions

1. In each case, find the partial fraction decomposition.

Answers: Page 24

(a) $\frac{1}{(x-1)(x-2)}$

(b) $\frac{2x+3}{x^2-x-6}$

(c) $\frac{5x^2-3x+2}{x^3-2x^2}$

(d) $\frac{7x^2-13x+13}{(x-2)(x^2-2x+3)}$

2. Integrate the following:

(a) $\int \frac{dx}{x^2+3x+2}$

(b) $\int \frac{2x+1}{(x+1)\left(x^2+1\right)}\,dx$

(c) $\int \frac{3x^2+7x-2}{x^3-x^2-2x}\,dx$

(d) $\int \frac{2x+3}{9x^2+6x+5}\,dx$

(e) $\int \frac{dx}{x^3+1}$

(f) $\int \frac{x^3+10x^2+3x+36}{(x-1)\left(x^2+4\right)^2}\,dx$

(g) $\int \frac{5x^3-4x^2+2x+1}{5x^2-4x-1}\,dx$

2.5 Challenge Integration Practice

1-5: Integrate the following:

Answers: Page 25

1. $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$
2. $\displaystyle\int x^5\sqrt{x^3+1}\,dx$
3. $\displaystyle\int \sec^3 x\,dx$
4. $\displaystyle\int_0^1 \sqrt{x^2+1}\,dx$
5. $\displaystyle\int e^{4x}\sqrt{1+e^{2x}}\,dx$

2.6 Improper Integrals

1-10: In each case, determine whether this integral converges or diverges. If it converges, evaluate the integral.

Answers: Page 25

1. $\displaystyle\int_1^\infty x^{-3}\,dx$
2. $\displaystyle\int_0^\infty e^{-3x}\,dx$
3. $\displaystyle\int_2^\infty \frac{dx}{x-1}$
4. $\displaystyle\int_{-3}^3 \frac{dx}{\sqrt{9-x^2}}$
5. $\displaystyle\int_0^2 f(x)\,dx$, where
$$f(x)=\begin{cases} x^{\frac{-1}{2}} & 0<x\le 1 \\ x-1 & 1<x\le 2 \end{cases}$$
6. $\displaystyle\int_0^\pi \tan\left(\frac{x}{3}\right)\,dx$
7. $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\,dx$
8. $\displaystyle\int_{-\infty}^\infty \frac{dx}{x^2+1}$
9. $\displaystyle\int_{-\infty}^\infty \frac{dx}{e^x+e^{-x}}$
10. $\displaystyle\int_{-1}^1 \ln|x|\,dx$

Module 3

Integration Applications

3.1 Review - Areas Between Curves

1. Find the area of the region R bound by the line $y = x$ and the parabola $y = 6 - x^2$.
2. Find the area of the region R enclosed by $y = \sin x$ and $y = \cos x$ from $x = 0$ to $x = 2\pi$.
3. Find the area of the region R enclosed by $y = 2x - 1$, $y = x^2 - 4$, $x = 1$, and $x = 2$.

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3.2 Volumes by Cross Sections

1. The region R is bounded by the curves $y = x^2$ and $y = 1$. R is rotated about the line $y = 2$, generating a ring shaped solid. Sketch the region R as well as a typical cross section of the solid. Find the volume of the solid.
2. Find the volume of the solid S obtained by rotating the region bounded by $y = x^2$ and $y = x^3$ about the x-axis.
3. Find the volume of the right-circular cone with base radius r and height h. Note: the cone is generated by rotating the triangle with vertices $(0, 0)$, $(0, h)$, and (r, h) about the y-axis.
4. Consider the region R, bound by $y = x^3$ and $y = \sqrt{x}$. Find the volume of the resulting solid if
   (a) R is revolved around the x-axis.
   (b) R is revolved around the y-axis.
   (c) R is revolved around the vertical line $x = -1$.
5. (a) Sketch the curve given by $x = 2y - y^2$.
   (b) Find the volume obtained by rotating the region enclosed by $x = 0$ and $x = 2y - y^2$ about the y-axis.
6. (a) R is bounded by $y = \sin x$, $y = 0$, $x = 0$, and $x = \pi$. Rotate R about the x-axis. Find the volume of the resulting solid.
   (b) R is bounded by $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \dfrac{\pi}{4}$. Rotate R about the x-axis. Find the volume of the resulting solid.

Answers: Page 25

7. A circular man-made lake has a 200m diameter and a maximum depth of 10m. Its cross section is the parabola $y = 10\left[\left(\frac{x}{100}\right)^2 - 1\right]$. Find the capacity of the lake.

3.3 Volumes by Cylindrical Shells

1. Consider the bowl obtained by revolving the region bounded by $y = x^2$, $y = 1$, and $x = 0$ about the y-axis.

Answers: Page 26

   (a) Find its volume using cross sections.
   (b) Find its volume using cylindrical shells.
   (c) Compare your answers.

2. Consider the region bounded by $y = x^3$ and $y = \sqrt{x}$. Use cylindrical shells to find the volume of the resulting solid if

   (a) R is revolved about the x-axis.
   (b) R is revolved about the y-axis.
   (c) R is revolved about the vertical line $x = -1$.
   Note: Compare your answers with those of question 4 in Section 3.2.

3. Find the volume V of the solid generated by revolving the region enclosed by $y = 3x^2 - x^3$, $y = 0$, $x = 0$, and $x = 3$ about the y-axis.

4. Determine the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x}$ and $y = x$ about the line $x = 5$.

5. Find the volume of the solid obtained by rotating the region bounded by $y = (x-1)^{\frac{1}{2}}$ and $y = (x-1)^2$ about the y-axis.

6. Consider the solid sphere of radius R. A cylinder of radius $r < R$ is bored through the center of the sphere. Find the volume of the remaining solid.

7. Consider $f(x) = \sin(x^2)$ and $g(x) = -\sin(x^2)$ from $x = 0$ to $x = \sqrt{\pi}$. Find the volume of revolution if the region enclosed by $f(x)$ and $g(x)$ is rotated about the y-axis.

8. Let R be the region in the first quadrant bounded by $y = (x-2)^{1/2}$ and let $y = 2$.

   (a) Find the resulting volume if R is rotated about the x-axis.
   (b) Find the resulting volume if R is rotated about the line $y = -2$.

3.4 Arclength

1. Find the length of the curve $f(x) = x^{\frac{3}{2}}$ between $x = 0$ and $x = 4$.

Answers: Page 26

2. Find the length of the curve $f(x) = 2e^x + \frac{1}{8}e^{-x}$ between $x = 0$ and $x = \ln 2$

3. Find the length of the curve $y = \left(\frac{3x}{2}\right)^{\frac{2}{3}} + 1$ between $x = 0$ and $x = \frac{2(3)^{\frac{3}{2}}}{3}$. Hint: consider the curve as a function $x(y)$ instead.

4. Find the length of the curve $x = \frac{1}{6}y^3 + \frac{1}{2y}$ between $y = 1$ and $y = 2$.

Module 4

Sequences and Series

4.1 Sequences

1-5: Determine whether the given sequence is convergent or divergent.

Answers: Page 26

1. $a_n = \dfrac{n^4 + 5n + 1}{n^4 + 2}$
2. $a_n = \dfrac{n^6 - 4n^2 + 3}{n^5 - 4n + 7}$
3. $a_n = e^{-n} \ln n$
4. $a_n = \ln\left(\dfrac{2n}{3n + 5}\right)$
5. $a_n = \dfrac{(3n)!}{(3n + 2)!}$

6-9: Determine whether the given sequence is increasing or decreasing. Also, determine whether it is bounded.

6. $a_n = \dfrac{2n}{5n + 3}$
7. $a_n = n^2 \left(4^{-n}\right)$
8. $a_n = \ln \dfrac{2n}{n + 5}$
9. $a_n = \dfrac{1}{3 + \ln n}$

4.2 Series

1-6: Determine whether the given series is convergent or divergent. If it is convergent, find its sum.

Answers: Page 27

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+3}{3n+4}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{2+3^n}{4^n}$
3. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{3^n} + \frac{3}{4\,(3^{n+2})} \right)$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{5}{(n+2)(n+3)}$
5. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{5n}{7n+4} \right)$
6. $\displaystyle\sum_{n=1}^{\infty} \left[\frac{3}{n(n+1)} - \frac{5}{n} \right]$

4.3 The Integral Test

1-6: Determine whether the given series is convergent or divergent.

Answers: Page 27

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^3}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{4n^2+3}$
3. $\displaystyle\sum_{n=1}^{\infty} n^3 e^{-2n^4}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{3\ln n}{n^4}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,(3\sqrt{n}+4)}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{4n^2+3}$

4.4 The Comparison Tests

1-6: Determine whether the given series is convergent or divergent.

Answers: Page 27

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+2n+5}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{16n^3+4n^2+3}}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^3-4n^2+5}{5n^6+3n^4+2n+1}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(5^n)}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{2+3^n}{5+6^n}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+2)^3}{2^n\,(1+3n+8n^3)}$

4.5 The Alternating Series Test

1-5: Determine whether the given series is convergent or divergent.

Answers: Page 27

1. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^3-3n+4}$
2. $\displaystyle\sum_{n=1}^{\infty} (-1)^n\, 2^{-n}$
3. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^2}$
4. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{4^n}$
5. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{e^{2n}}{n^3+1}$

4.6 Absolute Convergence and the Ratio and Root Tests

1-5: Determine whether the given series is convergent or divergent.

Answers: Page 27

1. $\sum_{n=1}^{\infty} \frac{4^n}{n(5^{n+1})}$
2. $\sum_{n=1}^{\infty} (-1)^n n e^{-n}$
3. $\sum_{n=1}^{\infty} \frac{n^n}{5^n}$
4. $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$
5. $\sum_{n=1}^{\infty} \frac{(3n)^n}{(2n+5)^n}$

4.7 Strategies for Testing Series

1-5: Determine whether the given series is convergent or divergent.

Answers: Page 27

1. $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$
2. $\sum_{n=1}^{\infty} \frac{5}{n(6^n)+3}$
3. $\sum_{n=1}^{\infty} \frac{\ln(2n)}{e^{2n}}$
4. $\sum_{n=1}^{\infty} \frac{(n+1)!}{n!\,2^n}$
5. $\sum_{n=1}^{\infty} \frac{(2n-5)^2}{5^n\,(6n^2+3n+2)}$

4.8 Power Series

1-5: Find the radius of convergence and the interval of convergence.

1. $\sum_{n=1}^{\infty} \frac{x^n}{n+3}$
2. $\sum_{n=1}^{\infty} \frac{4^n}{n^3} x^n$
3. $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n^2}$
4. $\sum_{n=1}^{\infty} \frac{(3x-5)^n}{5^{2n}}$
5. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+7}} (4x-5)^n$

Answers: Page 28

4.9 Representations of Functions as Power Series

1-5: Find a power series representation for $f(x)$ and determine the interval of convergence.

1. $f(x) = \frac{x}{2+x^2}$
2. $f(x) = \frac{4}{3x+7}$
3. $f(x) = \frac{x}{(2+3x)^2}$
4. $f(x) = \ln(x+3)$
5. $f(x) = \frac{3}{3+x^2}$

Answers: Page 28

4.10 Taylor and Maclaurin Series

1-3: Find the Maclaurin series for $f(x)$ and state the radius of convergence.

1. $f(x) = xe^{-3x}$
2. $f(x) = x^2 \sin x$
3. $f(x) = x \cos 4x$

Answers: Page 28

4-5: Find the Taylor series for $f(x)$ at the indicated number a and state the radius of convergence.

4. $f(x) = \sin x;\ a = \frac{\pi}{4}$
5. $f(x) = e^{2x};\ a = 4$

Module 5

Parameteric Equations and Polar Coordinates

5.1 Curves Defined by Parametric Equations

1-4: In the following problems (a) Sketch the graph of the curve having the indicated parametric equations, and (b) Eliminate the parameter to find a Cartesian equation of the curve.

Answers: Page 29

1. $x = t^2 + 2$, $y = t^2 - 3$; $-2 \leq t \leq 1$
2. $x = 4t^2 - 4$, $y = 2t + 5$; $t \in \mathbb{R}$
3. $x = 2\cos t$, $y = 2\sin t$; $0 \leq t \leq 2\pi$
4. $x = 4\cos t$, $y = 3\sin t$; $0 \leq t \leq 2\pi$

5.2 Calculus with Parametric Curves

1-3: Find the equation of the tangent line to the curve at the point corresponding to the given value of the parameter.

Answers: Page 29

1. $x = 4t^2 - 3$, $y = 3t + 3$; $t = 1$
2. $x = e^t$, $y = e^{-3t}$; $t = 0$
3. $x = 2\sin t$, $y = 4\cos t$; $t = \pi/4$

4-5: Find the points on the curve at which the tangent line horizontal or vertical.

4. $x = 4t^2$, $y = t^3 - 27t$
5. $x = e^t$, $y = t + e^{-2t}$

5.3 Polar Coordinates

1-4: Find a Cartesian equation for the given curve and identify it.

Answers: Page 29

1. $r = 4$
2. $r = 3\sin\theta$
3. $r^2\left(5\sin^2\theta - 4\cos^2\theta\right) = 2\theta$
4. $r^2\cos 2\theta = 4$

5-7: Find a polar equation that has the same graph as the given Cartesian equation.

5. $x^2 + y^2 = 25$
6. $x^2 - y^2 = 9$
7. $4x^2 + 25y^2 = 49$

8-10: Find the slope of the tangent line to the given curve at the indicated value of θ.

8. $r = 4\cos\theta$; $\theta = \pi/6$
9. $r = 1 + 2\cos\theta$; $\theta = \pi/3$
10. $r^2 = 2\cos 4\theta$; $\theta = \pi/4$

5.4 Areas and Lengths in Polar Coordinates

1-2: Sketch the graph of the equation and find the area of the region bounded by the graph.

Answers: Page 29

1. $r = 4\sin\theta$
2. $r = 1 + 2\cos\theta$

3-4: Find the area of the region bounded by one loop of the graph of the give equation.

3. $r = 6\cos 4\theta$
4. $r^2 = 2\cos 2\theta$

5-6: Find the area of the region that is outside the graph of the first equation and inside the graph of the second equation.

5. $r = 9$, $r = 6 + 6\cos\theta$
6. $r = 6 + 6\cos\theta$

7-8: Find the length of the given polar curve.

7. $r = e^{4\theta}$, $0 \leq \theta \leq 2$

8. $r = \cos^2\left(\frac{\theta}{2}\right)$, $0 \leq \theta \leq \pi$

Answers

1.1 Exercises (page 3)

1. **(a)** No **(b)** No **(c)** Yes **(d)** No **(e)** Yes
2. **(a)** $f^{-1}(x) = (x+1)^{\frac{1}{3}}$ **(b)** $f^{-1}(x) = (x+4)^{\frac{1}{2}} + 2$ **(c)** $f^{-1}(x) = \dfrac{1-2x}{x+1}$
3. **(a)** $\left(f^{-1}\right)'(8) = \dfrac{1}{14}$ **(b)** $\left(f^{-1}\right)'\left(\dfrac{1}{2}\right) = -\dfrac{2}{\sqrt{3}}$ **(c)** Not 1-1

1.2 Exercises (page 3)

1. **(a)** 6^x **(b)** $e^{\frac{4x}{3}}$
2. **(a)** $\ln\left(\dfrac{ex^4}{(3x)^{\frac{1}{2}}}\right)$ **(b)** $\log\left(\dfrac{100(x+1)^4}{x^2}\right)$
3. **(a)** $x = 5$ **(b)** $x = -3$ **(c)** $x = \dfrac{\ln(4)}{3}$ **(d)** $x = \dfrac{1}{4}(\log(2)+2) \approx 0.575$ **(e)** $x = e^e \approx 15.154$
4. **(a)** $k = \dfrac{\ln(2)}{53}$ **(b)** 8 Billion **(c)** In 2037
5. (a) 13300 years

1.3 Exercises (page 4)

1. **(a)** $f'(x) = 2xe^{x^2} + 12e^{4x}$ **(b)** $f''(x) = \left(\cos^2 x - \sin x\right)e^{\sin x}$ **(c)** $f'(x) = \dfrac{\frac{1}{2}x^{\frac{1}{2}}e^{x^{\frac{1}{2}}} - e^{x^{\frac{1}{2}}}}{x^2}$ **(d)** $f'(0) = 1$ **(e)** $f''(x) = \sec^2 x$ **(f)** $f'(x) = \dfrac{2}{x} - \dfrac{2x}{x^2+1}$
2. **(a)** $\dfrac{1}{4}e^{4x} + 4\ln|x| + c$ **(b)** $\dfrac{(e-1)^2}{e}$ **(c)** $\ln|x-1| + c$ **(d)** $\dfrac{1}{3}e^{x^3+3} + c$ **(e)** $\dfrac{3}{2}\ln\left|x^2+1\right| + c$ **(f)** $2e^2 - 2$
3. **(a)** $f'(x) = x^x(1+\ln(x))$ **(b)** $f'(x) = x^{\sin x}\left(\cos(x)\ln(x) + \dfrac{\sin x}{x}\right)$ **(c)** $f'(x) = \dfrac{(x-7)^5(2x+3)^{\frac{1}{2}}}{(x+1)^{\frac{1}{6}}}\left(\dfrac{5}{x-7} + \dfrac{1}{2x+3} - \dfrac{1}{6(x+1)}\right)$
4. $y = -\dfrac{2}{e}x + \dfrac{3}{e}$

5. $2e + \dfrac{2}{e} - 4$

1.4 Exercises (page 5)

1. **(a)** $\dfrac{\pi}{3}$ **(b)** $\dfrac{5\pi}{6}$ **(c)** $\dfrac{1}{2}$ **(d)** π
2. **(a)** $f'(x) = \dfrac{-3}{\sqrt{1-x^2}} + \dfrac{5}{1+x^2}$ **(b)** $f'(x) = \dfrac{3}{2\sqrt{3x}(1+3x)}$ **(c)** $f'(x) = \dfrac{-2x}{\sqrt{1-x^4}}$ **(d)** $f'(x) = \dfrac{-x}{\sqrt{1-x^2}}$
3. $y = \dfrac{x}{\sqrt{3}} + \dfrac{\pi}{3} - \dfrac{\pi\sqrt{3}}{9}$
4. **(a)** $\theta = \tan^{-1}\left(\dfrac{150}{x}\right)$ **(b)** $-0.0024\ \dfrac{\text{Rad}}{\text{ft}}$
5. **(a)** $\dfrac{\pi}{6}$ **(b)** $\dfrac{1}{2}\sin^{-1} x^2 + c$ **(c)** $\dfrac{1}{2}\left(\tan^{-1} x\right)^2 + c$

1.5 Exercises (page 6)

1. 0
2. $\dfrac{1}{4}$
3. 1
4. 0
5. 0
6. 1
7. 1
8. e
9. 0
10. 0

2.1 Exercises (page 7)

1. $-\dfrac{1}{2}x\cos 2x + \dfrac{1}{4}\sin 2x + c$
2. $-\dfrac{2}{3}xe^{-3x} - \dfrac{2}{9}e^{-3x} + c$
3. $3x^2\sin x + 6x\cos x - 6\sin x + c$
4. $\dfrac{1}{3}x^3\ln(x) - \dfrac{1}{9}x^3\Big]_1^e = \dfrac{2}{9}e^3 + \dfrac{1}{9}$
5. $(12x+20)\sin\left(\dfrac{x}{4}\right) + 48\cos\left(\dfrac{x}{4}\right)\Big]_0^{2\pi}$ $= 24\pi - 28$
6. $x\left(\ln(x)\right)^2 - 2x\ln(x) + 2x + c$
7. $\dfrac{2}{3}x(x+1)^{\frac{3}{2}} - \dfrac{4}{15}(x+1)^{\frac{5}{2}} + c$
8. $\dfrac{3}{13}\cos(2x)e^{3x} + \dfrac{2}{13}\sin(2x)e^{3x} + c$
9. $\dfrac{a}{b}x^2e^{bx} - \dfrac{2a}{b^2}xe^{bx} + \dfrac{2a}{b^3}e^{bx} + c$
10. $-x^5\cos x + 5x^4\sin x + 20x^3\cos x$ $- 60x^2\sin x - 120x\cos x + 120\sin x + c$

2.2 Exercises (page 7)

1. $\frac{1}{3}\sin^3 x - \frac{2}{5}\sin^5 x + \frac{1}{7}x + c$
2. $-\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + c$
3. $\frac{1}{8}x - \frac{1}{32}\sin 4x\Big]_0^{\pi} = \frac{\pi}{8}$
4. $\frac{3}{8}x + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$
5. $\frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + c$
6. $\frac{2}{3}\tan^{\frac{3}{2}} x + \frac{2}{7}\tan^{\frac{7}{2}} x + c$
7. $\frac{1}{6}\sec^3 2t + c$
8. $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x\Big]_0^{\frac{\pi}{2}} = \frac{8}{15}$
9. $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + \ln|\sec x| + c$

2.3 Exercises (page 8)

1. $\frac{1}{5}\left(1-x^2\right)^{\frac{5}{2}} - \frac{1}{3}\left(1-x^2\right)^{\frac{3}{2}} + c$
2. $-\frac{4}{3}\left(4-x^2\right)^{\frac{3}{2}} + \frac{4}{5}\left(4-x^2\right)^{\frac{5}{2}} + c$
3. $\tan(\theta) - \theta\Big]_0^{\frac{\pi}{3}} = \sqrt{3} - \frac{\pi}{3}$
4. $\frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{8}\cdot\frac{x}{4+x^2} + c$
5. $-2\sqrt{2-x^2} + \frac{1}{3}\left(2-x^2\right)^{\frac{3}{2}} + c$
6. $\frac{1}{16}\tan\theta\Big]_0^{\frac{\pi}{4}} = \frac{1}{16}$
7. $\frac{1}{4}\tan^{-1}\left(\frac{x-3}{2}\right) + c$
8. $3(\tan(\theta) - \theta)\Big]_0^{\frac{\pi}{3}} = 3\sqrt{3} - \pi$
9. $\ln\left|\frac{1}{2}\left(x^2-6x+13\right)^{\frac{1}{2}} + \frac{1}{2}(x-3)\right| + c$

2.4 Exercises (page 8)

1. **(a)** $\frac{1}{x-2} - \frac{1}{x-1}$ **(b)** $\frac{\frac{1}{5}}{x+2} + \frac{\frac{9}{5}}{x-3}$ **(c)** $\frac{1}{x} - \frac{1}{x^2} + \frac{4}{x-2}$ **(d)** $\frac{5}{x-2} + \frac{2x+1}{x^2-2x+3}$
2. **(a)** $\ln|x+1| - \ln|x+2| + c$
   (b) $-\frac{1}{2}\ln|x+1| + \frac{1}{4}\ln\left|x^2+1\right| + \frac{3}{2}\tan^{-1}x + c$
   (c) $\ln|x| - 2\ln|x+1| + 4\ln|x-2| + c$
   (d) $\frac{1}{9}\ln\left(9x^2+6x+5\right) + \frac{7}{18}\tan^{-1}\left(\frac{3x+1}{2}\right) + c$
   (e) $\frac{1}{3}\ln|x+1| - \frac{1}{6}\ln\left|x^2-x+1\right| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$
   (f) $2\ln|x-1| - \ln\left|x^2+4\right| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\cdot\frac{1}{x^2+4} + c$
   (g) $\frac{1}{2}x^2 - \frac{1}{15}\ln|5x+1| + \frac{2}{3}\ln|x-1| + c$

2.5 Exercises (page 9)

1. $\sin^{-1} e^x + c$ By substitution and trigonometric substitution
2. $\frac{2}{9}x^3\left(x^3+1\right)^{\frac{3}{2}} - \frac{4}{45}\left(x^3+1\right)^{\frac{5}{2}} + c$ By substitution and parts
3. $\frac{1}{2}\left(\tan(x)\sec(x) - \ln\left|\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right| + \ln\left|\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right|\right)$
4. $\frac{\sqrt{2} + \ln\left(\sqrt{2}+1\right)}{2}$ By parts and trigonometric substitution
5. $\frac{1}{5}\left(1+e^{2x}\right)^{\frac{5}{2}} - \frac{1}{3}\left(1+e^{2x}\right)^{\frac{3}{2}} + c$ By substitution and trigonometric substitution

2.6 Exercises (page 9)

1. Convergent $\frac{1}{2}$
2. Convergent $\frac{1}{3}$
3. Divergent
4. Convergent π
5. Convergent $\frac{5}{2}$
6. Convergent $3\ln(2)$
7. Convergent 0
8. Convergent π
9. Convergent $\frac{\pi}{2}$
10. Convergent -2

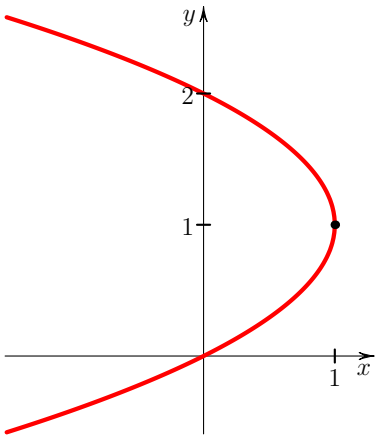
3.1 Exercises (page 11)

1. $\frac{125}{6}$
2. $4\sqrt{2}$
3. $\frac{11}{3}$

3.2 Exercises (page 11)

1. $\frac{56\pi}{15}$
2. $\frac{2\pi}{35}$
3. $\frac{1}{3}\pi r^2 h$
4. **(a)** $\frac{5\pi}{14}$ **(b)** $\frac{2\pi}{5}$ **(c)** $\frac{37\pi}{30}$

5. (a)



(b) $\frac{16\pi}{15}$

6. **(a)** $\frac{1}{2}\pi^2$ **(b)** $\frac{1}{2}\pi$

7. $50\ 000\pi$ m^3

3.3 Exercises (page 12)

1. **(a)** $V = \frac{\pi}{2}$ **(b)** $V = \frac{\pi}{2}$ **(c)** The same.
2. **(a)** $\frac{5\pi}{14}$ **(b)** $V = \frac{2\pi}{5}$ **(c)** $V = \frac{37\pi}{30}$
3. $\frac{243\pi}{10}$
4. $\frac{272\pi}{15}$
5. $\frac{29\pi}{30}$
6. $\frac{4\pi}{3}\left(R^2 - r^2\right)^{\frac{3}{2}}$
7. 4π
8. **(a)** 16π **(b)** $\frac{128\pi}{3}$

3.4 Exercises (page 12)

1. Length ≈ 9.1 units
2. Length $= \frac{33}{16}$ units
3. Length $= \frac{14}{3}$ units
4. Length $= \frac{17}{12}$ units

4.1 Exercises (page 13)

1. convergent
2. divergent
3. convergent
4. convergent
5. convergent
6. increasing, bounded
7. decreasing, bounded
8. increasing,bounded
9. decreasing,bounded

4.2 Exercises (page 14)

1. divergent
2. convergent, sum=$\frac{11}{3}$
3. convergent, sum= $\frac{25}{24}$
4. convergent, sum=$\frac{5}{3}$
5. divergent
6. divergent

4.3 Exercises (page 14)

1. convergent
2. divergent
3. convergent
4. convergent
5. divergent
6. convergent

4.4 Exercises (page 15)

1. convergent
2. divergent
3. convergent
4. convergent
5. convergent
6. convergent

4.5 Exercises (page 15)

1. convergent
2. convergent
3. convergent
4. convergent
5. divergent

4.6 Exercises (page 16)

1. convergent
2. convergent
3. divergent
4. convergent
5. divergent

4.7 Exercises (page 16)

1. convergent
2. convergent
3. convergent
4. convergent
5. convergent

4.8 Exercises (page 17)

1. $R = 1$, $-1 \leq x < 1$
2. $R = \frac{1}{4}$, $-\frac{1}{4} \leq x \leq \frac{1}{4}$
3. $R = 1$, $-3 \leq x \leq -1$
4. $R = \frac{25}{3}$, $-\frac{20}{3} < x < 10$
5. $R = \frac{1}{4}$, $1 \leq x < \frac{3}{2}$

4.9 Exercises (page 17)

1. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2^{n+1}}$, $|x| < \sqrt{2}$
2. $\displaystyle 4\sum_{n=0}^{\infty}(-1)^n \frac{3^n}{7^{n+1}}x^n$, $|x| < \frac{7}{3}$
3. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{3^{n+1}}{2^{n+1}}nx^n$, $|x| < \frac{2}{3}$
4. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{1}{3^{n+1}}\frac{x^{n+1}}{n+1} + \ln 3$, $|x| < 3$
5. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{3^n}$, $|x| < \sqrt{3}$

4.10 Exercises (page 17)

1. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{3^n}{n!}x^{n+1}$, $R = \infty$
2. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+3}}{(2n+1)!}$, $R = \infty$
3. $\displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{4^{2n}}{(2n)!}x^{2n+1}$, $R = \infty$
4. $\frac{\sqrt{2}}{2}\left[1 + \left(x - \frac{\pi}{4}\right) - \frac{1}{2!}\left(x - \frac{\pi}{4}\right)^2 - \frac{1}{3!}\left(x - \frac{\pi}{4}\right)^3 + \frac{1}{4!}\left(x - \frac{\pi}{4}\right)^4 + \cdots\right]$, $R = \infty$
5. $e^8\left[1 + 2\,(x-4) + \frac{4}{2!}\,(x-4)^2 + \frac{8}{3!}\,(x-4)^3 + \cdots\right]$, $R = \infty$

5.1 Exercises (page 19)

1. line segment: $y = x - 5$, $3 \leq x \leq 6$
2. parabola: $x = (y-5)^2 - 4$
3. circle: $x^2 + y^2 = 4$
4. ellipse: $\frac{x^2}{16} + \frac{y^2}{9} = 1$

5.2 Exercises (page 19)

1. $y = \frac{1}{4}x + \frac{23}{4}$
2. $y = -3x + 4$
3. $y = -2x + 4\sqrt{2}$
4. Horizontal if $t = \pm 3$, Vertical if $t = 0$
5. Horizontal if $t = \frac{\ln 2}{2}$, No Vertical

5.3 Exercises (page 20)

1. circle: $x^2 + y^2 = 16$
2. circle: $x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$
3. hyperbola: $\frac{y^2}{4} - \frac{x^2}{5} = 1$
4. hyperbola: $y^2 - x^2 = 4$
5. $r = 5$
6. $r^2 \cos(2\theta) = 9$
7. $r^2\left(4 + 21\sin^2\theta\right) = 49$
8. $\frac{dy}{dx} = -1$
9. $\frac{dy}{dx} = \frac{1}{3\sqrt{3}}$
10. $\frac{dy}{dx} = -1$

5.4 Exercises (page 20)

1. $A = 8\pi$
2. $A = 3\pi$
3. $A = \frac{9\pi}{8}$
4. $A = 1$
5. $A = \frac{81\sqrt{3}}{2} - 9\pi$
6. $A = \frac{81\sqrt{3}}{2} + 18\pi$
7. $L = \frac{\sqrt{17}}{4}\left(e^8 - 1\right)$
8. $L = 2$

Appendix A

References

The problems for this manual were collected from a variety of sources, including instructors's personal class notes and exams, as well as the following resources:

Adams, Essex: Calculus: A Complete Course, 8th Edition, Pearson.

Briggs, Chocran: Calculus: Early Transcendentals, Addison Wesley.

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