

Math 221-001 201710
Assignment # 1

Due: January 20th, 2017

1. Construct the truth table for the following statements:

- (a) $\sim (P \wedge Q)$
- (b) $(\sim P) \vee (\sim Q)$
- (c) $\sim (P \wedge Q) \Rightarrow (\sim P) \vee (\sim Q)$

2. Show, by using truth tables, the following logical equivalences:

- (a) $\sim (P \Rightarrow Q) \equiv (P \wedge (\sim Q))$
- (b) $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$; $\sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q)$
- (c) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$; $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$
- (d) $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$; $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
- (e) $P \Rightarrow Q \equiv (\sim Q) \Rightarrow (\sim P)$

3. Assume each of the following statements are true. Use that information to determine the truth values of each of P, Q, R, S, U, V .

$$P \vee Q, Q \Rightarrow R, P \wedge S \Rightarrow V, \sim R, \sim Q \Rightarrow U \wedge S$$

4. An *argument* is a statement of the form $(P_1 \wedge P_2 \wedge \dots \wedge P_n) \Rightarrow Q$. We say that the argument is *valid* if it is a tautology. It is customary to write the argument in the form

$$\begin{array}{c} P_1 \\ P_2 \\ \vdots \\ P_n \\ \hline Q \end{array} .$$

The statements P_1, \dots, P_n are the *assumptions* and Q is the *conclusion*.

(a) Show that the following arguments are valid.

- i. (Disjunctive syllogism) $\frac{P \vee Q}{\sim P} \frac{Q}{Q}$
- ii. (Modus Tollens) $\frac{\sim Q}{P \Rightarrow Q} \frac{\sim P}{\sim P}$
- iii. (Hypothetical syllogism) $\frac{P \Rightarrow Q}{Q \Rightarrow R} \frac{P \Rightarrow R}{P \Rightarrow R}$
- iv. (Resolution) $\frac{P \vee Q}{\sim P \vee R} \frac{Q \vee R}{Q \vee R}$

(b) Write each of the following arguments formally.

- i. I go for a run or I watch tv. I didn't go for a run. Therefore, I watch tv.
- ii. If it snows a lot, the university will close. The university is not closed. Therefore, it did not snow.

(c) Decide whether the following argument is valid. If it is, then give a proof; if it is not, explain why.

When Alex goes fishing, his friends George and Daniel go too. Since Daniel is friends with Luke, Luke's presence at the dock is a sufficient condition for him go as well. On the other hand, for Daniel to go fishing it is necessary that Alex also be there (as he needs someone to talk to during the boring hours). Therefore, Luke won't go fishing unless George also goes.