Math 221-001 201710

Assignment # 1 - Answers

1. Construct the truth table for the following statements:

   (a) $\sim (P \wedge Q)$

   (b) $(\sim P) \vee (\sim Q)$

   (c) $\sim (P \wedge Q) \Rightarrow (\sim P) \vee (\sim Q)$

   Answer.

(a)

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

(b)

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(c)

P	Q	$\sim (P \wedge Q)$	$\sim P \vee \sim Q$	$\sim (P \wedge Q) \Rightarrow \sim P \vee \sim Q$
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T

2. Show, by using truth tables, the following logical equivalences:

   (a) $\sim (P \Rightarrow Q) \equiv (P \wedge (\sim Q))$

   (b) $\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q)$; $\sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q)$

   (c) $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$; $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

   (d) $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$; $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

   (e) $P \Rightarrow Q \equiv (\sim Q) \Rightarrow (\sim P)$

   Answer. (selected)

(a)

P	Q	$P \Rightarrow Q$	$\sim (P \Rightarrow Q)$	$P \wedge (\sim Q)$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

3. Assume each of the following statements are true. Use that information to determine the truth values of each of P, Q, R, S, U, V.

$$P \vee Q, Q \Rightarrow R, P \wedge S \Rightarrow V, \sim R, \sim Q \Rightarrow U \wedge S$$

Answer.

Since $\sim R$ is true, we know that R is FALSE.

Now, since $Q \implies R$ is true and R is false, Q is FALSE.

$P \vee Q$ is true and Q is false, so P is TRUE.

$\sim Q \implies (U \wedge S)$ is true and $\sim Q$ is true, so $(U \wedge S)$ is TRUE.

Since $(U \wedge S)$ is true, U is TRUE , S is TRUE.

Since $P \wedge S \implies V$ is true, and P and S are true, $P \wedge S$ is true, and so V is TRUE.

4. An *argument* is a statement of the form $(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \implies Q$. We say that the argument is *valid* if it is a tautology. It is customary to write the argument in the form

$$\begin{array}{c} P_1 \\ P_2 \\ \vdots \\ P_n \\ \hline Q \end{array} .$$

The statements $P_1, \ldots, P_n$ are the *assumptions* and Q is the *conclusion*.

(a) Show that the following arguments are valid.

i. (Disjunctive syllogism) $\begin{array}{c} P \vee Q \\ \sim P \\ \hline Q \end{array}$

Answer. We have

P	Q	$P \vee Q$	$\sim P$	$(P \vee Q) \wedge (\sim P)$	$(P \vee Q) \wedge (\sim P) \implies Q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

ii. (Modus Tollens) $\begin{array}{c} \sim Q \\ P \implies Q \\ \hline \sim P \end{array}$

Answer. We could of course use a truth table, but let us reason in a different way. The argument is of the form $(P_1 \wedge P_2) \implies Q$. If the antecedent in the implication is false, the implication will be automatically true. So we only need to consider the case where $P_1 \wedge P_2$ is true, which is turn only happens when both P_1 and P_2 are true. In summary, what we can do it to assume that P_1, P_2 are true, and deduce that Q is true; that would guarantee that the implication is always true and thus the argument is valid.

So we assume $\sim Q$ and $P \implies Q$ to be true. If $\sim Q$ is true, then Q is false. As $P \implies Q$ is true, this forces P to be false, because otherwise the implication would be of the form $T \implies F$. As P is false, $\sim P$ is true.

iii. (Hypothetical syllogism) $\begin{array}{l} P \implies Q \\ \underline{Q \implies R} \\ P \implies R \end{array}$

Answer. Here we assume that the two implications $P \implies Q$ and $Q \implies R$ are true. If P is false, we automatically get that $P \implies R$ is true. If P is true, then Q is true (otherwise, $P \implies Q$ would be false). Since Q is true and so is $Q \implies R$, it follows that R is true. Thus both P and R are true, and $P \implies R$ is true.

iv. (Resolution) $\begin{array}{l} P \vee Q \\ \underline{\sim P \vee R} \\ Q \vee R \end{array}$

Answer. We assume that $P \vee Q$ and $\sim P \vee R$ are true. If Q is true, then $Q \vee R$ is true. If Q is false, then P has to be true to make $P \vee Q$ true. Then $\sim P$ is false; as $\sim P \vee R$ is true, we get that R is true. Thus $Q \vee R$ is true.

(b) Write each of the following arguments formally.

i. I go for a run or I watch tv. I didn't go for a run. Therefore, I watch tv.
Answer.

$$\begin{array}{l} P \vee Q \\ \underline{\sim P} \\ Q \end{array}$$

ii. If it snows a lot, the university will close. The university is not closed. Therefore, it did not snow.
Answer.

$$\begin{array}{l} P \implies Q \\ \underline{\sim Q} \\ \sim P \end{array}$$

(c) Decide whether the following argument is valid. If it is, then give a proof; if it is not, explain why.

When Alex goes fishing, his friends George and Daniel go too. Since Daniel is friends with Luke, Luke's presence at the dock is a sufficient condition for him go as well. On the other hand, for Daniel to go fishing it is necessary that Alex also be there (as he needs someone to talk to during the boring hours). Therefore, Luke won't go fishing unless George also goes.

Answer. We can symbolize the argument in the following way: let

$A =$ "Alex goes fishing",
$D =$ "Daniel goes fishing",
$G =$ "George goes fishing",
$L =$ "Luke goes fishing".

Then the argument goes

$$\begin{array}{ccc} A & \implies & G \wedge D \\ L & \implies & D \\ D & \implies & A \\ \hline \sim G & \implies & \sim L \end{array}$$

Using the contrapositive,

$$\begin{array}{ccc} A & \implies & G \wedge D \\ L & \implies & D \\ D & \implies & A \\ \hline L & \implies & G \end{array}$$

Now there are several ways to check that this argument is valid. One would be a truth table, showing that on every row where the first three implications are true, the last one is also true. The annoying fact is that the truth table would have 16 rows.

Another way is the following: since $L \implies D$ and $D \implies A$, we conclude (hypothetical syllogism) that $L \implies A$; as $A \implies (G \wedge D)$, again syllogism gives us $L \implies (G \wedge D)$. As this last implication is true, so is $L \implies G$, because otherwise we would have L and $\sim G$, which makes $L \implies (G \wedge D)$ false.

Yet another way: for contradiction, assume that $L \implies G$ is false. Then L is true and G is false. Thus $G \wedge D$ is false, and the implication $A \implies (G \wedge D)$ true forces A to be false. With A false, the implication $D \implies A$ forces D to be false, and in a similar way the implication $L \implies D$ forces then L to be false. But this is a contradiction, since we were assuming that L was true. Then $L \implies G$ is true.