

Math 221-001 201710
Assignment # 2

Due: January 27th, 2017

1. Let A, B, C be subsets of \mathcal{U} . Prove

- (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;
- (b) $(A \cap B)^c = A^c \cup B^c$;

2. Indicate “true” or “false” in each of the following assertions. Explain.

- (a) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
- (b) $\emptyset \subset \{\emptyset, \{\emptyset\}\}$
- (c) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (d) $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$
- (e) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
- (f) $\{a, b\} \in \{\{a, b, c\}, \{a, c\}, a, b\}$
- (g) $\{a, b\} \subset \{\{a, b, c\}, \{a, c\}, a, b\}$
- (h) $\emptyset \in \{\{a, b, c\}, \{a, c\}, a, b\}$
- (i) $\emptyset \subset \{\{a, b, c\}, \{a, c\}, a, b\}$
- (j) $\emptyset \in \mathcal{P}(\{1, 2\})$.

3. Let A, B, C be sets. Determine if the following assertions are true or false. Provide a proof or a counterexample in each case.

- (a) $(A \cap B) \subset (B \cap C) \Rightarrow A \subset B$.
- (b) $(A \cup B) \subset (B \cup C) \Rightarrow A \subset B$.
- (c) $\mathcal{P}(C) \setminus \mathcal{P}(B) \subset \mathcal{P}(B \setminus C)$.
- (d) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
- (e) $\mathcal{P}(A) \cup \mathcal{P}(B) \subset \mathcal{P}(A \cup B)$; this one is true: prove it, and find a counterexample for the case of equality.

4. For each of the following, give a proof or counterexample. Write also the negation of each statement.

- (a) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x + y > 5$
- (b) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y > 5$
- (c) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x - y > 3$
- (d) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x - y > 3$
- (e) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy > 2$
- (f) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy > 2$