

Math 221-001 201710
Assignment # 3

Due: February 3rd, 2017

(Recall that all assertions about the integers should be proven using the Postulates and/or results that we obtained using the postulates)

1. Show that the multiplicative identity for the integers is unique.
2. Note that the list of postulates on the web page, which we will use, requires that $0 \neq 1$ by definition. Show that if we remove that requirement (i.e. the way the Postulates were written in the blackboard in class) then the set $\{0\}$ consisting of 0 alone, with the operations $0 + 0 = 0$, $0 \cdot 0 = 0$, satisfies all the Postulates.
3. Prove that if $a, b, c \in \mathbb{Z}$ and $a + b = a + c$, then $b = c$.
4. Let $a, b \in \mathbb{Z}$. Show that
 - (a) If $a > 0$, $b > 0$, then $ab > 0$;
 - (b) if $a > 0$, $b < 0$, then $ab < 0$;
 - (c) if $a < 0$, $b < 0$, then $ab > 0$.
5. Prove that if $a, b \in \mathbb{Z}$ with $a, b \neq 0$, then $ab \neq 0$.
6. Prove that if $a, b, c \in \mathbb{Z}$ with $a \neq 0$ and $ab = ac$, then $b = c$.