Math 221-001 201710
Assignment # 4

Due: February 10th, 2017

1. Prove (by contradiction) that $\sqrt{1+\sqrt{2}}$ is irrational (you can use the fact that $\sqrt{2}$ is irrational; this makes the proof much simpler that the one we did for $\sqrt{2}$).

2. Let k be an integer. Prove

   (a) k is odd if and only if k^2 is odd;

   (b) $k^2 + k$ is even;

   (c) if k is odd, then 8 divides $k^2 - 1$;

   (d) the product of any three consecutive integers, of which the middle one is odd, is divisible by 8.

3. Use induction to show that $n^3 + 2n$ is a multiple of 3 for every integer $n \geq 1$. Then prove that $n^3 + 2n$ is a multiple of 3 for every $n \in \mathbb{Z}$.

4. Prove that for all integers $n \geq 4$, $n^2 \leq 2^n$.

5. Prove that for all integers $n \geq 1$, $n^3 - n$ is a multiple of 6.

6. Prove that, for every integer $n \geq 1$,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$