

Math 221-001 201710  
Assignment # 5

**Due: March 3rd, 2017**

1. Let  $a, b, c, d \in \mathbb{Z}$ . Prove

- (a) if  $a|b$  and  $c|d$ , then  $ac|bd$ ;
- (b) if  $ab|c$ , then  $a|c$ ;
- (c) if  $a|b$  and  $b|c$  then  $a|c$ ;
- (d) if  $a|b$  and  $a|c$  then  $a|(b + c)$ .

2. For each  $a, b$  find the unique  $q, r \in \mathbb{Z}$  such that  $a = qb + r$ ,  $0 \leq r < b$ .

- (a)  $a = 256, b = 4$ ;
- (b)  $a = -256, b = 4$ ;
- (c)  $a = 255, b = 4$ ;
- (d)  $a = -255, b = 4$ .

3. Find the greatest common divisor of 252 and -180, and also integers  $m$  and  $n$  such that it can be written as  $252m - 180n$ .

4. Find the smallest integer in the set

$$\{x \in \mathbb{Z}^+ : x = 6s + 15t \text{ for some } s, t \in \mathbb{Z}\},$$

and also integers  $m$  and  $n$  such that it can be written as  $6m + 15n$ .

5. Find the smallest integer of the set

$$\{x \in \mathbb{Z}^+ : x = 3780p + 1200q \text{ for some } p, q \in \mathbb{Z}\},$$

and also integers  $m$  and  $n$  such that it can be written as  $3780p + 1200q$ .

6. Let  $a, b, m, n \in \mathbb{Z}$  such that  $ma + nb = 1$ . Show that  $\gcd(a, b) = 1$ .

7. Find the general solution of the following Diophantine equations:

- (a)  $212x + 37y = 1$ ;
- (b)  $91x + 126y = 203$ ;
- (c)  $169x - 65y = 91$ .

8. Can 120 be expressed as the sum of two positive integers, one of which is divisible by 11 and the other by 17?