

Math 221-001 201710
Assignment # 7

Due: March 17th 2017; *there will be no lecture that day, so the assignments should be handed in at the math department, CW307.14*

1. Solve the following simultaneous congruences.

$$(a) \begin{cases} x \equiv 46 \pmod{51} \\ x \equiv 27 \pmod{52} \end{cases}$$

$$(b) \begin{cases} 2x \equiv 11 \pmod{13} \\ 3x \equiv 7 \pmod{10} \\ 7x \equiv 5 \pmod{8} \end{cases}$$

2. Find all integers in the range 1000-4000 satisfying the simultaneous congruences:

$$\begin{cases} x \equiv 2 \pmod{7} \\ x \equiv 5 \pmod{11} \\ x \equiv 11 \pmod{17} \end{cases}$$

3. Prove that $21 \mid (3n^7 + 7n^3 + 11n)$ for every integer n .

4. Let $\phi(m)$ be the Euler phi-function. Show that

(a) $\phi(m) = \phi(2m)$ if and only if m is odd.

(b) $\phi(m) = m - 1$ if and only if m is prime.

5. Find the remainder when $2^{(2^{100})}$ is divided by 29.