

Math 221-001 201710  
Assignment # 8

**Due:** March 31st.

1. Show that inclusion is an order relation in the set of all sets.
2. For the following sets and relations, determine whether they are equivalence relations. If they are, determine all equivalence classes.
  - (a)  $X = \{1, 2, 3\}$ , and  $R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ .
  - (b)  $X = \{1, 2, 3\}$ , and  $S = \{(1, 1), (2, 2), (3, 3)\}$ .
  - (c)  $\mathbb{Z}$ , and  $R = \{(n, m) : n = m + 1\}$ .
  - (d)  $\mathbb{Z}$ , and  $R = \{(n, m) : 3|n - m\}$ .
3. List all possible equivalence relations on the set  $\{a, b, c\}$ .
4. Find a relation  $R$ , on a set  $S$ , that is symmetric and transitive, but not reflexive.
5. The following argument shows that if a relation is symmetric and transitive, then it is reflexive. If you did question 4 right, then you know this has to be false. Find the error in the argument

“Assume that  $R$  is symmetric and transitive on  $S$ . For any  $a, b \in S$ ,  $aRb$  implies  $bRa$ , because  $R$  is symmetric. Then, from  $aRb$  and  $bRa$  we conclude by transitivity  $aRa$ . Since  $aRa$ ,  $R$  must be reflexive.”