Math 221-001 201710
Assignment # 9

Due: April 7th.

1. Below we write H for the set of all human beings. Decide if each of the following relations is a function.
   (a) $A = \{(t, s) :\ s^2 = t\} \subset \mathbb{R} \times \mathbb{R}$;
   (b) $B = \{(t, s) :\ t^2 = s\} \subset \mathbb{R} \times \mathbb{R}$;
   (c) $C = \{(x, y) :\ x \text{ is the mother of } y\} \subset H \times H$;
   (d) $D = \{(x, y) :\ x \text{ is the daughter of } y\} \subset H \times H$;
   (e) $E = \{(a, b) :\ a \text{ is the age of } b\} \subset \mathbb{Z} \times H$;
   (f) $F = \{(a, b) :\ b \text{ is the age of } a\} \subset H \times \mathbb{Z}$.

2. Let $f : A \to B$ and $g : B \to C$, such that f is injective and g is surjective. Is $g \circ f$ surjective? Is it injective? Provide proof/counterexample.

3. Let $f : A \to B$ and $g : B \to C$, such that f is surjective and g is injective. Is $g \circ f$ surjective? Is it injective? Provide proof/counterexample.

4. Find an example of functions $f : A \to B$ and $g : B \to C$ such that $g \circ f$ is surjective but f is not surjective.

5. Let $f : A \to B$. Prove the following:
   (a) If f is invertible, then f is bijective;
   (b) If f is bijective, then f is invertible.

6. The identity function $\mathrm{id}_X : X \to X$ is sometimes denoted by $1_X : X \to X$ (that is, $1_X(x) = x$ for every $x \in X$). Find an example of sets X, Y and functions $f : X \to Y$ and $g : Y \to X$ such that $g \circ f = 1_X$ but $f \circ g \neq 1_Y$.

7. Consider, for each pair of integers m and n, the function $f_{m,n} : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $f_{m,n}(x, y) = mx + ny$ (that is, different choices of m and n give you different functions). Decide which values of m and n (if any) make $f_{m,n}$ surjective, and which values of m and n (if any) make $f_{m,n}$ injective.