Math 122-002 201730
Practice Assignment # 3

1. The scores of three players in a tournament have been lost. The only information available is that the sum of score for players 1 and 2 is 20, the sum of the scores of players 2 and 3 is 15, and the sum of the scores for players 1 and 3 is 17.

   (a) Show that the individual scores can be recovered. Find them!

   (b) If we had four players, and we know the combined scores of 1 and 2, 2 and 3, 3 and 4, and 4 and 1, can the scores be recovered?

2. Find a such that the system has nontrivial solutions. $\left\{\begin{array}{rcrcrcl} & x & + & 2y & + & z & = & 0 \\ & x & + & ay & - & 3z & = & 0 \\ - & x & + & 6y & - & 5z & = & 0 \end{array}\right.$

3. Without solving the system ("by inspection"), determine whether each homogeneous system has nontrivial (that is, nonzero) solutions. Justify your answers.

   (a) $\left\{\begin{array}{rcrcrcrcl} 2x_1 & + & 2x_2 & + & 2x_3 & + & 5x_4 & = & 0 \\ -x_1 & + & 5x_2 & + & 2x_3 & - & 6x_4 & = & 0 \\ 8x_1 & + & x_2 & + & 4x_3 & + & 1000x_4 & = & 0 \end{array}\right.$

   (b) $\left\{\begin{array}{rcrcrcl} x_1 & + & 5x_2 & - & 2x_3 & = & 0 \\ & & 5x_2 & + & 8x_3 & = & 0 \\ & & & & 2x_3 & = & 0 \end{array}\right.$

   (c) $\left\{\begin{array}{rcrcl} 2x_1 & + & 2x_2 & = & 0 \\ -x_1 & - & x_2 & = & 0 \end{array}\right.$

4. Suppose that A, B, C, D are matrices of sizes 3×4, 2×3, 3×3 and 2×4 respectively. Decide whether the given expression is defined; if it is, give the size of the resulting matrix

   (a) AB
   (b) BA
   (c) $BA + C$
   (d) BD
   (e) DAC
   (f) $D + B$

5. The *transpose* of an $m \times n$ matrix A is the matrix A^T obtained by interchanging the rows and columns of A. This means that the first row of A^T is the first column of A, etc. In symbols, $(A^T)_{ij} = A_{ji}$.

Now consider the matrices

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 7 \\ 5 & 2 & 0 \\ 1 & 2 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 6 & 3 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix}$$

and compute the given expression where possible:

(a) $AB + C^T$

(b) $(BA)^T$

(c) ACB

(d) $(B + C)A$

(e) $A(B + C)$