Math 122-002 201730
Practice Assignment # 5 – Answers

1. Compute AB using the indicated block partitioning

$$A = \left[\begin{array}{cc|cc} 2 & -1 & 3 & 1 \\ 1 & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right], \qquad B = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ -1 & 0 & 0 \\ \hline 0 & 5 & 1 \\ 1 & -1 & 0 \end{array}\right].$$

Answer. Writing in blocks,

$$A = \begin{bmatrix} X & Y \\ 0_{2\times 2} & I_2 \end{bmatrix}, \quad B = \begin{bmatrix} Z & \vec{0} \\ W & \vec{x} \end{bmatrix},$$

where

$$X = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}, \; Y = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}, \; Z = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \; W = \begin{bmatrix} 0 & 5 \\ 1 & -1 \end{bmatrix}, \; \vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

In block form,

$$AB = \begin{bmatrix} XZ + YW & X\vec{0} + Y\vec{x} \\ W & \vec{x} \end{bmatrix} = \begin{bmatrix} XZ + YW & Y\vec{x} \\ W & \vec{x} \end{bmatrix}.$$

Computations:

$$XZ + YW = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 14 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 18 \\ 3 & 5 \end{bmatrix}$$

$$Y\vec{x} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

Thus

$$AB = \begin{bmatrix} 4 & 18 & 3 \\ 3 & 5 & 1 \\ 0 & 5 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

2. Assuming all blocks are $k \times k$ (in particular $I = I_{k\times k}$), compute:

(a) $\begin{bmatrix} I & X \\ 0 & I \end{bmatrix}^2$

(b) $\begin{bmatrix} I & X \\ -Y & I \end{bmatrix} \begin{bmatrix} I & 0 \\ Y & I \end{bmatrix}$

(c) $\begin{bmatrix} I & X \end{bmatrix} \begin{bmatrix} I & X \end{bmatrix}^T$

(d) $\begin{bmatrix} I & X \end{bmatrix}^T \begin{bmatrix} I & X \end{bmatrix}$.

Answer.

(a)

$$\begin{bmatrix} I & X \\ 0 & I \end{bmatrix}^2 = \begin{bmatrix} I & X \\ 0 & I \end{bmatrix} \begin{bmatrix} I & X \\ 0 & I \end{bmatrix} = \begin{bmatrix} I & 2X \\ 0 & I \end{bmatrix}.$$

(b)

$$\begin{bmatrix} I & X \\ -Y & I \end{bmatrix} \begin{bmatrix} I & 0 \\ Y & I \end{bmatrix} = \begin{bmatrix} I + XY & X \\ 0 & I \end{bmatrix}.$$

(c)

$$\begin{bmatrix} I & X \end{bmatrix} \begin{bmatrix} I & X \end{bmatrix}^T = \begin{bmatrix} I & X \end{bmatrix} \begin{bmatrix} I \\ X^T \end{bmatrix} = I + XX^T.$$

(d)

$$\begin{bmatrix} I & X \end{bmatrix}^T \begin{bmatrix} I & X \end{bmatrix} = \begin{bmatrix} I \\ X^T \end{bmatrix} \begin{bmatrix} I & X \end{bmatrix} = \begin{bmatrix} I & X \\ X^T & X^T X \end{bmatrix}$$

3. Solve the system by finding the inverse of a matrix.

(a) $\begin{cases} 3x - 4y = 8 \\ -x + \ \ y = 9 \end{cases}$ (b) $\begin{cases} x + y = 2 \\ x - y = 3 \end{cases}$

Answer.

(a) We have $A = \begin{bmatrix} 3 & -4 \\ -1 & 1 \end{bmatrix}$, and so

$$A^{-1} = \frac{1}{3-4} \begin{bmatrix} 1 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ -1 & -3 \end{bmatrix}.$$

Then

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\vec{b} = \begin{bmatrix} -1 & -4 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix} = \begin{bmatrix} -8 - 36 \\ -8 - 27 \end{bmatrix} = \begin{bmatrix} -44 \\ -35 \end{bmatrix}$$

So $x = -44$, $y = -35$ is the solution.

(b) We have $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, and so

$$A^{-1} = \frac{1}{-1-1} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}.$$

Then

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\vec{b} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5/2 \\ -1/2 \end{bmatrix}$$

So $x = 5/2$, $y = -1/2$ is the solution.

4. Let

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Calculate A^2, and use the result to find A^{-1}.

Answer. We have

$$A^2 = \begin{bmatrix} 0+0+1 & 0+0+0 & 0+0+0 \\ 0+1-1 & 0+1+0 & 1-1+0 \\ 0+0+0 & 0+0+0 & 1+0+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$