Math 122-002 201730
Practice Assignment # 9

Please remember that the assignment consists of only a sample of the kind of questions you are supposed to be able to do. It is **not** *a safe practice to just do the assignment, and that is why there is a list of "suggested practice problems".*

1. Consider the vectors $\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ in $\mathbb{R}^2$. Draw the vectors

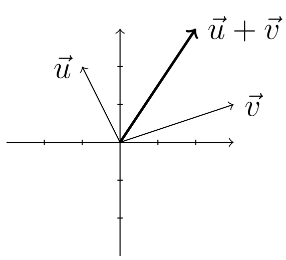
   (a) $\vec{u} + \vec{v}$;

   (b) $2\vec{u}$ and $(-1/2)\vec{w}$;

   (c) $2(\vec{u} + \vec{w})$;

   (d) $\vec{v} + \vec{u} - \vec{w}$.

Answer.

(a) 

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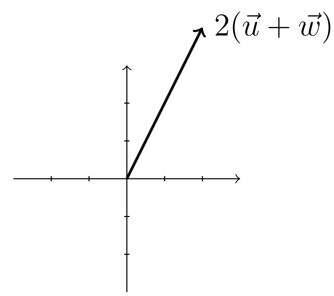
(c) 

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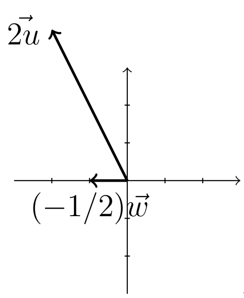
(b) 

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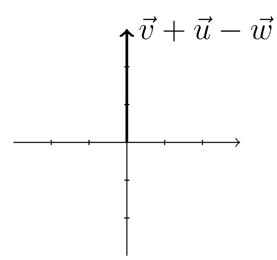
(d) 

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2. Consider the following vectors:

$$\vec{r} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \quad \vec{s} = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix} \quad \text{in } \mathbb{R}^2;$$

$$\vec{u} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \quad \text{in } \mathbb{R}^3;$$

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 6 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 3 \end{bmatrix} \quad \text{in } \mathbb{R}^4.$$

(a) Find $\vec{u}+\vec{v}-\vec{w}$ and $\vec{x}-\vec{y}$.

(b) Find $\sqrt{2}\,\vec{s}-2\vec{r}$, $3\vec{u}$, $-\vec{r}$, and $0\vec{x}+1\vec{y}$.

(c) Find $\|\vec{u}\|$, $\|\vec{v}\|$, $\|\vec{u}+\vec{v}\|$.

(d) Find $\|\vec{s}\|$, $\|\vec{r}\|$, $\|\vec{x}\|$, $\|\vec{y}\|$.

(e) Find $\vec{s}\cdot\vec{r}$, $\vec{u}\cdot\vec{v}$, $\vec{x}\cdot\vec{y}$.

(f) Decide if $\vec{x}$ and $\vec{y}$ are orthogonal.

(g) Find $\vec{u}\times\vec{v}$.

Answer.

(a)

$$\vec{u}+\vec{v}-\vec{w}=\begin{bmatrix}3\\2\\-1\end{bmatrix}+\begin{bmatrix}3\\-4\\5\end{bmatrix}-\begin{bmatrix}0\\0\\3\end{bmatrix}=\begin{bmatrix}6\\-2\\1\end{bmatrix}.$$

$$\vec{x}-\vec{y}=\begin{bmatrix}1\\2\\0\\6\end{bmatrix}-\begin{bmatrix}-1\\0\\2\\3\end{bmatrix}=\begin{bmatrix}2\\2\\-2\\3\end{bmatrix}.$$

(b)

$$\sqrt{2}\,\vec{s}-2\vec{r}=\sqrt{2}\begin{bmatrix}1.5\\0\end{bmatrix}-2\begin{bmatrix}-1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}=\begin{bmatrix}3\sqrt{2}/2+\sqrt{2}\\-\sqrt{2}\end{bmatrix}=\left[5\sqrt{2}/2,-\sqrt{2}\right].$$

$$3\vec{u}=3\begin{bmatrix}3\\2\\-1\end{bmatrix}=\begin{bmatrix}9\\6\\-3\end{bmatrix}.$$

$$-\vec{r}=\begin{bmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{bmatrix}.$$

$$0\vec{x}+1\vec{y}=\vec{y}=\begin{bmatrix}-1\\0\\2\\3\end{bmatrix}.$$

(c)

$$\|\vec{u}\|=\sqrt{3^2+2^2+(-1)^2}=\sqrt{9+4+1}=\sqrt{14}.$$

$$\|\vec{v}\|=\sqrt{3^2+4^2+5^2}=\sqrt{9+16+25}=\sqrt{50}.$$

$$\|\vec{u}+\vec{v}\|=\sqrt{(3+3)^2+(2-4)^2+(-1+5)^2}=\sqrt{36+4+16}=\sqrt{56}.$$

(d)

$$\|\vec{s}\|=\sqrt{1.5^2+0^2}=1.5;\ \|\vec{r}\|=\sqrt{\frac{1}{2}+\frac{1}{2}}=1.$$

$$\|\vec{x}\|=\sqrt{1^2+2^2+0^2+6^2}=\sqrt{1+4+36}=\sqrt{41}.$$

$$\|\vec{y}\|=\sqrt{(-1)^2+0^2+2^2+3^2}=\sqrt{1+4+9}=\sqrt{14}.$$

(e)

$$\vec{s}\cdot\vec{r} = \frac{-1.5}{\sqrt{2}} + 0 = -\frac{3}{2\sqrt{2}};$$
$$\vec{u}\cdot\vec{v} = 3\times 3 + 2\times(-4) + (-1)\times 5 = 9 = 8 - 5 = -4.$$
$$\vec{x}\cdot\vec{y} = -1 + 0 + 0 + 18 = 17.$$

(f) We have just shown that $\vec{x}\cdot\vec{y} \neq 0$, so they are **not** orthogonal.

(g)

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & -1 \\ 3 & -4 & 5 \end{vmatrix} = (6, -18, -18).$$

3. Calculate $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$, $\|\vec{u}\|$, $\|\vec{v}\|$, $5\vec{v}$, $-\sqrt{2}\vec{u}$.

   (a) $\vec{u} = 2\check{\mathbf{i}} - 3\check{\mathbf{j}} + 5\check{\mathbf{k}}$, $\vec{v} = \sqrt{2}\check{\mathbf{i}} + 6\check{\mathbf{j}} - 5\check{\mathbf{k}}$
   Answer. $\vec{u}+\vec{v} = (2+\sqrt{2})\check{\mathbf{i}} + 3\check{\mathbf{j}}$; $\vec{u}-\vec{v} = (2-\sqrt{2})\check{\mathbf{i}} - 9\check{\mathbf{j}} + 10\check{\mathbf{k}}$;
   $\|\vec{u}\| = \sqrt{2^2+(-3)^2+5^2} = \sqrt{38}$; $\|\vec{v}\| = \sqrt{(\sqrt{2})^2+6^2+(-5)^2} = \sqrt{2+36+25} = \sqrt{63}$.
   $5\vec{v} = 5\sqrt{2}\check{\mathbf{i}} + 30\check{\mathbf{j}} - 25\check{\mathbf{k}}$; $-\sqrt{2}\vec{u} = -2\sqrt{2}\check{\mathbf{i}} + 3\sqrt{2}\check{\mathbf{j}} - 5\sqrt{2}\check{\mathbf{k}}$.

   (b) $\vec{u} = 2\check{\mathbf{i}} - 5\check{\mathbf{j}}$, $\vec{v} = \check{\mathbf{i}} + 5\check{\mathbf{j}} - \check{\mathbf{k}}$
   Answer. $\vec{u}+\vec{v} = 3\check{\mathbf{i}} - \check{\mathbf{k}}$; $\vec{u}-\vec{v} = \check{\mathbf{i}} - 10\check{\mathbf{j}} + \check{\mathbf{k}}$;
   $\|\vec{u}\| = \sqrt{2^2+(-5)^2} = \sqrt{29}$; $\|\vec{v}\| = \sqrt{1^2+5^2+(-1)^2} = \sqrt{27} = 3\sqrt{3}$;
   $5\vec{v} = 5\check{\mathbf{i}} + 25\check{\mathbf{j}} - 5\check{\mathbf{k}}$; $-\sqrt{2}\vec{u} = -2\sqrt{2}\check{\mathbf{i}} + 5\sqrt{2}\check{\mathbf{j}}$.

   (c) $\vec{u} = \check{\mathbf{i}} + \check{\mathbf{j}} + \check{\mathbf{k}}$, $\vec{v} = 2\check{\mathbf{i}} - 2\check{\mathbf{j}} + 3\check{\mathbf{k}}$.
   Answer. $\vec{u}+\vec{v} = 3\check{\mathbf{i}} - \check{\mathbf{j}} + 4\check{\mathbf{k}}$; $\vec{u}-\vec{v} = -\check{\mathbf{i}} + 3\check{\mathbf{j}} - 2\check{\mathbf{k}}$;
   $\|\vec{u}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$; $\|\vec{v}\| = \sqrt{2^2+(-2)^2+3^2} = \sqrt{17}$;
   $5\vec{v} = 10\check{\mathbf{i}} - 10\check{\mathbf{j}} + 15\check{\mathbf{k}}$; $-\sqrt{2}\vec{u} = -\sqrt{2}\check{\mathbf{i}} - \sqrt{2}\check{\mathbf{j}} - \sqrt{2}\check{\mathbf{k}}$.

4. Compute the dot product of the two vectors. Determine the cosine of the angle between them, and answer whether they are orthogonal or not

   (a) $\check{\mathbf{i}}$, $2\check{\mathbf{i}} - 3\check{\mathbf{j}} + \check{\mathbf{k}}$
   Answer. $\check{\mathbf{i}}\cdot(2\check{\mathbf{i}} - 3\check{\mathbf{j}} + \check{\mathbf{k}}) = 1\times 2 + 0\times(-3) + 0\times 1 = 2$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{2}{\sqrt{1}\sqrt{4+9+1}} = \frac{2}{\sqrt{14}},$$

   so $\cos\theta \neq 0$ (i.e. $\theta \neq \pi/2$). The two vectors are **not** orthogonal.

   (b) $-4\check{\mathbf{i}} - 3\check{\mathbf{j}} + 4\check{\mathbf{k}}$, $-8\check{\mathbf{i}} - 5\check{\mathbf{j}} + \check{\mathbf{k}}$
   Answer. $(-4\check{\mathbf{i}} - 3\check{\mathbf{j}} + 4\check{\mathbf{k}})\cdot(-8\check{\mathbf{i}} - 5\check{\mathbf{j}} + \check{\mathbf{k}}) = 32 + 15 + 4 = 51.$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{51}{\sqrt{16+9+16}\sqrt{64+25+1}} = \frac{51}{\sqrt{3690}},$$

   so $\cos\theta \neq 0$ (i.e. $\theta \neq \pi/2$). The two vectors are **not** orthogonal.

(c) $\check{\mathbf{i}} - 3\check{\mathbf{k}}$, $2\check{\mathbf{j}} + 6\check{\mathbf{k}}$

Answer. $(\check{\mathbf{i}} - 3\check{\mathbf{k}}) \cdot (2\check{\mathbf{j}} + 6\check{\mathbf{k}}) = -18$.

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|} = \frac{-18}{\sqrt{1+9}\sqrt{4+36}} = -\frac{18}{20},$$

so $\cos\theta \neq 0$ (i.e. $\theta \neq \pi/2$). The two vectors are **not** orthogonal.

5. What can you say about the vector $\vec{v}$ if

(a) $\vec{v} \cdot \vec{v} = 0$?

Answer. If $\vec{v} = a\check{\mathbf{i}} + b\check{\mathbf{j}} + c\check{\mathbf{k}}$ and $\vec{v} \cdot \vec{v} = 0$, then

$$0 = \vec{v} \cdot \vec{v} = a^2 + b^2 + c^2.$$

This implies that $a = b = c = 0$, so $\vec{v} = \bar{0}$.

(b) $\vec{v} \cdot \check{\mathbf{i}} = 0$, $\vec{v} \cdot \check{\mathbf{j}} = 0$, $\vec{v} \cdot \check{\mathbf{k}} = 0$?

Answer. We have $a = \vec{v} \cdot \check{\mathbf{i}} = 0$, $b = \vec{v} \cdot \check{\mathbf{j}} = 0$, $c = \vec{v} \cdot \check{\mathbf{k}} = 0$, so $a = b = c = 0$ again, and $\vec{v} = \bar{0}$.