Math 122-002 201730
Practice Assignment # 10 – Answers

Please remember that the assignment consists of only a sample of the kind of questions you are supposed to be able to do. It is **not** *a safe practice to just do the assignment, and that is why there is a list of "suggested practice problems".*

1. Find parametric equations of the following lines. Write also the vector form.
   (a) The line containing $(1, 0, 4)$, $(2, 1, 1)$.
   (b) The line containing $(2, 1, 1)$, $(2, 1, -2)$.
   (c) The line containing $(-4, -2, 5)$, $(1, 1, -5)$.
   (d) The line parallel to $\vec{v} = (0, 1, 2)$ and passing through $(1, 2, 3)$.
   (e) The line parallel to $\vec{v} = (0, 1, 2)$ and passing through $(0, 1, 2)$.
   (f) The line parallel to $\vec{v} = (1, 2, 3)$ and passing through $(1, 2, 3)$.
   (g) The line parallel to $\vec{v} = (1, 2, 3)$ and passing through $(0, 1, 2)$.

   Answer.

   (a) the equation of a line is $\vec{x} = A + t(B - A)$. So the parametric form is

$$\begin{aligned} x &= 1 + t(2-1) = 1 + t \\ y &= 0 + t(1-0) = t \\ z &= 4 + t(1-4) = 4 - 3t \end{aligned}$$

   Vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}.$$

   (b) The parametric equations are

$$\begin{aligned} x &= 2 \\ y &= 1 \\ z &= -2 + 3t \end{aligned}$$

   Vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}.$$

   (c)

$$\begin{aligned} x &= 1 + 5t \\ y &= 1 + 3t \\ z &= -5 - 10t \end{aligned}$$

   Vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix} + t \begin{bmatrix} 5 \\ 3 \\ -10 \end{bmatrix}$$

(d)

$$\begin{aligned} x &= 1 \\ y &= 2 + t \\ z &= 3 + 2t \end{aligned}$$

Vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(e)

$$\begin{aligned} x &= 0 \\ y &= 1 + t \\ z &= 2 + 2t \end{aligned}$$

Vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(f)

$$\begin{aligned} x &= 1 + t \\ y &= 2 + 2t \\ z &= 3 + 3t \end{aligned}$$

Vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(g)

$$\begin{aligned} x &= t \\ y &= 1 + 2t \\ z &= 2 + 3t \end{aligned}$$

Vector form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

2. Find the points of intersection (if any) of the following lines:

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 11 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

Answer. By equating the two expressions, we get a system

$$\begin{cases} -2s + t = 10 - 3 \\ -s - t = 1 - 0 \\ -s + 5t = 11 - 3 \end{cases}$$

Using row-reduction,

$$\begin{bmatrix} -2 & 1 & | & 7 \\ -1 & -1 & | & 1 \\ -1 & 5 & | & 8 \end{bmatrix} \xrightarrow[-R1]{R1 \leftrightarrow R3} \begin{bmatrix} 1 & -5 & | & -8 \\ -2 & 1 & | & 7 \\ -1 & -1 & | & 1 \end{bmatrix} \xrightarrow[R2+2R1]{R3+R1} \begin{bmatrix} 1 & -5 & | & -8 \\ 0 & -9 & | & -9 \\ 0 & -6 & | & -7 \end{bmatrix}$$

$$\xrightarrow[R3+6R2]{-R2/9} \begin{bmatrix} 1 & -5 & | & -8 \\ 0 & 1 & | & 1 \\ 0 & 0 & | & 1 \end{bmatrix}$$

and we see that the system is inconsistent. So the two lines do not intersect.

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 14 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

Answer. By equating the two expressions, we get a system

$$\begin{cases} -2s + t = 10 - 3 \\ -s - t = 1 - 0 \\ -s + 5t = 14 - 3 \end{cases}$$

Using row-reduction,

$$\begin{bmatrix} -2 & 1 & | & 7 \\ -1 & -1 & | & 1 \\ -1 & 5 & | & 11 \end{bmatrix} \xrightarrow[-R1]{R1 \leftrightarrow R2} \begin{bmatrix} 1 & 1 & | & -1 \\ -2 & 1 & | & 7 \\ -1 & 5 & | & 11 \end{bmatrix} \xrightarrow[R2+2R1]{R3+R1} \begin{bmatrix} 1 & 1 & | & -1 \\ 0 & 3 & | & 5 \\ 0 & 6 & | & 10 \end{bmatrix}$$

$$\xrightarrow[R2/3]{R2-2R2} \begin{bmatrix} 1 & 1 & | & -1 \\ 0 & 1 & | & 5/3 \\ 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{R1-R2} \begin{bmatrix} 1 & 0 & | & -8/3 \\ 0 & 1 & | & 5/3 \\ 0 & 0 & | & 0 \end{bmatrix}$$

So $s = -8/3$, $t = 5/3$, and we get the point

$$\begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + \frac{5}{3} \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 14/3 \\ -5/3 \\ 34/3 \end{bmatrix}.$$

To confirm, we can also obtain it with s, as

$$\begin{bmatrix} 10 \\ 1 \\ 14 \end{bmatrix} - \frac{8}{3} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 14/3 \\ -5/3 \\ 34/3 \end{bmatrix}.$$

The two lines intersect at $(14/3, -5/3, 34/3)$.

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix} + s \begin{bmatrix} -2 \\ 2 \\ -10 \end{bmatrix}$

Answer. By equating the two expressions, we get a system

$$\begin{cases} 2s + t = 4 - 3 \\ -2s - t = -1 - 0 \\ 10s + 5t = 8 - 3 \end{cases}$$

Using row-reduction,

$$\begin{bmatrix} 2 & 1 & | & 1 \\ -2 & -1 & | & -1 \\ 10 & 5 & | & 5 \end{bmatrix} \xrightarrow[R3-5R2]{R2+R1} \begin{bmatrix} 2 & 1 & | & 1 \\ 0 & 0 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

The system has infinitely many solutions, which means that the two lines are the same.