University of Regina
Department of Mathematics and Statistics
Math 122-002 201730
Midterm 1 – Answers

1. Find the Reduced Row Echelon Form and solve the system

$$\begin{cases} 2x_1 &+& 2x_2 &+& 2x_3 &+& 4x_4 &=& 0 \\ -x_1 &+& 5x_2 &+& 2x_3 & & &=& 2 \\ 8x_1 &+& x_2 &+& 4x_3 &+& 2x_4 &=& -2 \end{cases}$$

Answer. We perform row reduction:

$$\left[\begin{array}{rrrr|r} 2&2&2&4&0\\ -1&5&2&0&2\\ 8&1&4&2&-2 \end{array}\right] \xrightarrow[\substack{R2+R1\\ R3-8R1}]{\frac12\times R1} \left[\begin{array}{rrrr|r} 1&1&1&2&0\\ 0&6&3&2&2\\ 0&-7&-4&-14&-2 \end{array}\right] \xrightarrow[-R2]{R2+R3} \left[\begin{array}{rrrr|r} 1&1&1&2&0\\ 0&1&1&12&0\\ 0&-7&-4&-14&-2 \end{array}\right]$$

$$\xrightarrow[R3+7R2]{R1-R2} \left[\begin{array}{rrrr|r} 1&0&0&-10&0\\ 0&1&1&12&0\\ 0&0&3&70&-2 \end{array}\right] \xrightarrow[\frac13 R3]{R2-\frac13 R3} \left[\begin{array}{rrrr|r} 1&0&0&-10&0\\ 0&1&0&-34/3&2/3\\ 0&0&1&70/3&-2/3 \end{array}\right]$$

This gives us, taking $x_4 = t$ as a parameter,

$$x_1 = 10t, \quad x_2 = \frac{2}{3} + \frac{34t}{3}, \quad x_3 = -\frac{2}{3} - \frac{70t}{3}, \quad x_4 = t.$$

Comments. *Note that fractions were avoidable until almost the very end; this will usually help to avoid computational mistakes. There were several instances of things like "$2R1 + R2$"; this is not illegal in row reduction, but it is an almost sure source of mistakes because you are doing your calculations with numbers that are not in the page (twice each element of R1). A number of students stopped before achieving the RREF, which was a specific requirement in this question.*

2. Find the value of b that makes the system consistent

$$\begin{cases} 2x + 3y = 1 \\ x + y = 2 \\ x + 2y = b \end{cases}$$

Answer. Performing row reduction,

$$\left[\begin{array}{rr|r} 2&3&1\\ 1&1&2\\ 1&2&b \end{array}\right] \xrightarrow{R1\leftrightarrow R2} \left[\begin{array}{rr|r} 1&1&2\\ 2&3&1\\ 1&2&b \end{array}\right] \xrightarrow[R3-R1]{R2-2R1} \left[\begin{array}{rr|r} 1&1&2\\ 0&1&-3\\ 0&1&b-2 \end{array}\right] \xrightarrow{R3-R2} \left[\begin{array}{rr|r} 1&1&2\\ 0&1&-3\\ 0&0&b+1 \end{array}\right]$$

So for the system to be consistent we need $b+1=0$; that is, $b=-1$.

Comments. *A very quick method that several students noticed is to subtract the third equation from the first one; then we get $x+y=1-b$, while the second row gives $x+y=2$. So $1-b=2$, and then $b=-1$. Or subtract the second from the first one to get $x+2y=-1$, so comparing with the third row we get $b=x+2y=-1$. This works fine in this particular problem, but in general it might not work, while row reduction will always work.*

3. Calculate $(BA)^T$, where

$$A=\begin{bmatrix}2 & 0\\ 1 & 3\\ 3 & -1\end{bmatrix},\quad B=\begin{bmatrix}4 & 1 & 7\\ 5 & 2 & 0\\ 1 & 2 & 5\end{bmatrix}.$$

Answer. We have

$$BA=\begin{bmatrix}4 & 1 & 7\\ 5 & 2 & 0\\ 1 & 2 & 5\end{bmatrix}\times\begin{bmatrix}2 & 0\\ 1 & 3\\ 3 & -1\end{bmatrix}=\begin{bmatrix}4\times 2+1\times 1+7\times 3 & 4\times 0+1\times 3+7\times(-1)\\ 5\times 2+2\times 1+0\times 3 & 5\times 0+2\times 3+0\times(-1)\\ 1\times 2+2\times 1+5\times 3 & 1\times 0+2\times 3+5\times(-1)\end{bmatrix}=\begin{bmatrix}30 & -4\\ 12 & 6\\ 19 & 1\end{bmatrix}$$

Then

$$(BA)^T=\begin{bmatrix}30 & 12 & 19\\ -4 & 6 & 1\end{bmatrix}.$$

Comments. *An answer needs to be written in a way that makes sense. Several students wrote things like*

$$BA=\begin{bmatrix}30 & -4\\ 12 & 6\\ 19 & 1\end{bmatrix}^T=\begin{bmatrix}30 & 12 & 19\\ -4 & 6 & 1\end{bmatrix}$$

This makes no sense: BA is a 3×2 matrix, while $(BA)^T$ is a 2×3 matrix; they are not equal, so they cannot be related by equal signs! On the other hand, this would have been correct:

$$(BA)^T=\begin{bmatrix}30 & -4\\ 12 & 6\\ 19 & 1\end{bmatrix}^T=\begin{bmatrix}30 & 12 & 19\\ -4 & 6 & 1\end{bmatrix}$$

4. Let

$$\vec{a_1}=\begin{bmatrix}2\\ -1\\ 8\end{bmatrix},\quad \vec{a_2}=\begin{bmatrix}2\\ 5\\ 1\end{bmatrix},\quad \vec{a_3}=\begin{bmatrix}2\\ 2\\ 4\end{bmatrix},\quad \vec{a_4}=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix},\quad \vec{b}=\begin{bmatrix}0\\ 2\\ -2\end{bmatrix}.$$

Write $\vec{b}$ as a linear combination of $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, $\vec{a}_4$ **in two different ways**. *(Helpful hint: you can use the results of Question 1).*

Answer. We use the solution from question 1. Taking $t=0$ we get $x_1=0$, $x_2=2/3$, $x_3=-2/3$, $x_0=0$. So

$$\vec{b}=\frac{2}{3}\,\vec{a}_2-\frac{2}{3}\,\vec{a}_3.$$

For another choice of coefficients we use a different value of t. For instance if $t = 1$, we get $x_1 = 10$, $x_2 = 36/3 = 12$, $x_3 = -72/3 = -24$, $x_4 = 1$. Thus

$$\vec{b} = 10\vec{a}_1 + 12\vec{a}_2 - 24\vec{a}_3 + \vec{a}_4.$$