

University of Regina
Department of Mathematics and Statistics
Math 122-002 201730
Quiz # 1 - Answers

Both versions of the quiz were very similar. An answer is provided first for the "M" version and then for the "H" version (difference was only in swapping of rows).

1. (version M) Find the reduced row echelon form of the system, and use it to find all solutions to the system.

$$\begin{cases} 4x_1 + 2x_2 + 7x_3 + 13x_4 = 1 \\ 6x_1 + 3x_2 + 9x_3 + 18x_4 = 2 \\ 2x_1 + x_2 + 3x_3 + 4x_4 = 0 \end{cases}$$

Answer. Working on the augmented matrix of the system,

$$\begin{aligned} & \left[\begin{array}{cccc|c} 4 & 2 & 7 & 13 & 1 \\ 6 & 3 & 9 & 18 & 2 \\ 2 & 1 & 3 & 4 & 0 \end{array} \right] \xrightarrow{\substack{R3 \leftrightarrow R1 \\ R2 \leftrightarrow R3}} \left[\begin{array}{cccc|c} 2 & 1 & 3 & 4 & 0 \\ 4 & 2 & 7 & 13 & 1 \\ 6 & 3 & 9 & 18 & 2 \end{array} \right] \xrightarrow{\substack{R2-2R1 \\ R3-3R1}} \left[\begin{array}{cccc|c} 2 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 6 & 2 \end{array} \right] \\ & \xrightarrow{\substack{R1-3R2 \\ \frac{1}{6}R3}} \left[\begin{array}{cccc|c} 2 & 1 & 0 & -11 & -3 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{\substack{R1+11R3 \\ R2-5R3}} \left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 2/3 \\ 0 & 0 & 1 & 0 & -2/3 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{\frac{1}{2}R1} \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 1/3 \\ 0 & 0 & 1 & 0 & -2/3 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \end{aligned}$$

And we have obtained the reduced row echelon form. In terms of the system, we have

$$\begin{cases} x_1 + \frac{1}{2}x_2 = 1/3 \\ x_3 = -2/3 \\ x_4 = 1/3 \end{cases}$$

The only column without a leading unit is the second one. So, using $x_2 = t$ as a parameter, all solutions to the system can be expressed by

$$\begin{cases} x_1 = 1/3 - t/2 \\ x_2 = t \\ x_3 = -2/3 \\ x_4 = 1/3 \end{cases}$$

1. (version H) Find the reduced row echelon form of the system, and use it to find all solutions to the system.

$$\begin{cases} 6x_1 + 3x_2 + 9x_3 + 18x_4 = 2 \\ 4x_1 + 2x_2 + 7x_3 + 13x_4 = 1 \\ 2x_1 + x_2 + 3x_3 + 4x_4 = 0 \end{cases}$$

Answer. Working on the augmented matrix of the system,

$$\begin{aligned} & \left[\begin{array}{cccc|c} 6 & 3 & 9 & 18 & 2 \\ 4 & 2 & 7 & 13 & 1 \\ 2 & 1 & 3 & 4 & 0 \end{array} \right] \xrightarrow{R3 \leftrightarrow R1} \left[\begin{array}{cccc|c} 2 & 1 & 3 & 4 & 0 \\ 4 & 2 & 7 & 13 & 1 \\ 6 & 3 & 9 & 18 & 2 \end{array} \right] \xrightarrow{\begin{array}{l} R2-2R1 \\ R3-3R1 \end{array}} \left[\begin{array}{cccc|c} 2 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 6 & 2 \end{array} \right] \\ & \xrightarrow{\begin{array}{l} R1-3R2 \\ \frac{1}{6}R3 \end{array}} \left[\begin{array}{cccc|c} 2 & 1 & 0 & -11 & -3 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{\begin{array}{l} R1+11R3 \\ R2-5R3 \end{array}} \left[\begin{array}{cccc|c} 2 & 1 & 0 & 0 & 2/3 \\ 0 & 0 & 1 & 0 & -2/3 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \xrightarrow{\frac{1}{2}R1} \left[\begin{array}{cccc|c} 1 & 1/2 & 0 & 0 & 1/3 \\ 0 & 0 & 1 & 0 & -2/3 \\ 0 & 0 & 0 & 1 & 1/3 \end{array} \right] \end{aligned}$$

And we have obtained the reduced row echelon form. In terms of the system, we have

$$\begin{cases} x_1 + \frac{1}{2}x_2 = 1/3 \\ x_3 = -2/3 \\ x_4 = 1/3 \end{cases}$$

The only column without a leading unit is the second one. So, using $x_2 = t$ as a parameter, all solutions to the system can be expressed by

$$\begin{cases} x_1 = 1/3 - t/2 \\ x_2 = t \\ x_3 = -2/3 \\ x_4 = 1/3 \end{cases}$$