Name: ____________________ H
Student ID: ____________________

University of Regina
Department of Mathematics and Statistics
Math 122-002 201730
Quiz # 4

Date: November 16^{th}, 2017

Duration: 20 minutes

Instructions: Submit well-written solutions to the problems below. Please be sure to write your name and student number on the front. Use the back of the page if you need more space. Complete details are required for full credit. The quiz consists of one question.

1. Diagonalize A and use this information to find A^8. Helpful fact: $2^8 = 256$.

$$A = \begin{bmatrix} -5 & 7 \\ -6 & 8 \end{bmatrix}.$$

Answer. We first find the eigenvalues: the characteristic polynomial is

$$\det(A - \lambda I) = \begin{vmatrix} -5-\lambda & 7 \\ -6 & 8-\lambda \end{vmatrix} = (\lambda+5)(\lambda-8) + 42 = \lambda^2 - 3\lambda + 2 = (\lambda-2)(\lambda-1).$$

So the eigenvalues are $2, 1$.

Eigenvectors for $\lambda = 2$: We need to solve the homogeneous system with matrix $A - 2I$. So we row reduce

$$\begin{bmatrix} -5-2 & 7 \\ -6 & 8-2 \end{bmatrix} = \begin{bmatrix} -7 & 7 \\ -6 & 6 \end{bmatrix} \xrightarrow{-R1/7,\ R2+6R1} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Solutions will be $x = y$, which we write in vector form as $t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Eigenvectors for $\lambda = 1$: we now solve with $A - I$:

$$\begin{bmatrix} -5-1 & 7 \\ -6 & 8-1 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ -6 & 7 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} -6 & 7 \\ 0 & 0 \end{bmatrix}.$$

Solutions will be $7y = 6x$, in vector form $s \begin{bmatrix} 7 \\ 6 \end{bmatrix}$.

We then have

$$P = \begin{bmatrix} 1 & 7 \\ 1 & 6 \end{bmatrix}.$$

This matrix has determinant equal to -1, so we obtain the inverse directly as

$$P^{-1} = \begin{bmatrix} -6 & 7 \\ 1 & -1 \end{bmatrix}.$$

Then

$$A^{16} = \begin{bmatrix} 1 & 7 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 2^8 & 0 \\ 0 & 1^8 \end{bmatrix} \begin{bmatrix} -6 & 7 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 256 & 7 \\ 256 & 6 \end{bmatrix} \begin{bmatrix} -6 & 7 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1529 & 1785 \\ -1530 & 1786 \end{bmatrix}$$

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1. Diagonalize A and use this information to find A^6. Helpful fact: $3^6 = 729$.

$$A = \begin{bmatrix} 9 & -6 \\ 8 & -5 \end{bmatrix}.$$

Answer. We first find the eigenvalues: the characteristic polynomial is

$$\det(A - \lambda I) = \begin{vmatrix} 9-\lambda & -6 \\ 8 & -5-\lambda \end{vmatrix} = (\lambda+5)(\lambda-9) + 48 = \lambda^2 - 4\lambda + 3 = (\lambda-3)(\lambda-1).$$

So the eigenvalues are $3, 1$.

Eigenvectors for $\lambda = 3$: We need to solve the homogeneous system with matrix $A - 2I$. So we row reduce

$$\begin{bmatrix} 9-3 & -6 \\ 8 & -5-3 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ 8 & -8 \end{bmatrix} \xrightarrow{R1/6,\ R2-8R1} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Solutions will be $x = y$, which we write in vector form as $t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Eigenvectors for $\lambda = 1$: we now solve with $A - I$:

$$\begin{bmatrix} 9-1 & -6 \\ 8 & -5-1 \end{bmatrix} = \begin{bmatrix} 8 & -6 \\ 8 & -6 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} 8 & -6 \\ 0 & 0 \end{bmatrix}.$$

Solutions will be $6y = 8x$, in vector form $s \begin{bmatrix} 6 \\ 8 \end{bmatrix}$.

We then have

$$P = \begin{bmatrix} 1 & 6 \\ 1 & 8 \end{bmatrix}.$$

This matrix has determinant equal to 2, and obtain the inverse directly as

$$P^{-1} = \frac{1}{2} \begin{bmatrix} 8 & -6 \\ -1 & 1 \end{bmatrix}.$$

Then

$$A^6 = \frac{1}{2} \begin{bmatrix} 1 & 6 \\ 1 & 8 \end{bmatrix} \begin{bmatrix} 3^6 & 0 \\ 0 & 1^6 \end{bmatrix} \begin{bmatrix} 8 & -6 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 729 & 6 \\ 729 & 8 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 2913 & -2184 \\ 2912 & -2183 \end{bmatrix}$$