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University of Regina
Department of Mathematics and Statistics
Math 122-002 201730
Quiz # 5 – Answers

1. Let $\vec{v} = \check{\mathbf{i}} - 2\check{\mathbf{j}} + 3\check{\mathbf{k}}$ and $\vec{w} = 3\check{\mathbf{i}} - 2\check{\mathbf{j}} + \check{\mathbf{k}}$.

   (a) Write the parametric equation of the following lines:

   i. the line parallel to $\vec{v} - 2\vec{w}$ and passing through $(2, 2, 2)$

   ii. the line parallel to $\vec{v} + 2\vec{w}$ and passing through $(-5, 8, -3)$

   (b) Find, if any, the intersection points of the two lines above.

Answer. We calculate

$$\vec{v} - 2\vec{w} = \begin{bmatrix} 1 - 2 \times 3 \\ -2 - 2 \times (-2) \\ 3 - 2 \times 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix},$$

$$\vec{v} + 2\vec{w} = \begin{bmatrix} 1 + 2 \times 3 \\ -2 + 2 \times (-2) \\ 3 + 2 \times 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \\ 5 \end{bmatrix}$$

The lines are then

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + t \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}, \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \\ -3 \end{bmatrix} + s \begin{bmatrix} 7 \\ -6 \\ 5 \end{bmatrix}.$$

In parametric form:

$$\begin{aligned} x &= 2 - 5t \\ y &= 2 + 2t \\ z &= 2 + t \end{aligned} \qquad \text{and} \qquad \begin{aligned} x &= -5 + 7s \\ y &= 8 - 6s \\ z &= -3 + 5s \end{aligned}$$

For the intersection, we need to solve the system

$$\begin{aligned} 2 - 5t &= -5 + 7s \\ 2 + 2t &= 8 - 6s \\ 2 + t &= -3 + 5s \end{aligned}$$

which we can write as

$$\begin{aligned} -5t - 7s &= -7 \\ 2t + 6s &= 6 \\ t - 5s &= -5 \end{aligned}$$

Doing row reduction,

$$\begin{bmatrix} -5 & 7 & | & -7 \\ 2 & 6 & | & 6 \\ 1 & -5 & | & -5 \end{bmatrix} \xrightarrow[R2-2R3]{R1+5R3} \begin{bmatrix} 0 & -18 & | & -18 \\ 0 & 16 & | & 16 \\ 1 & -5 & | & -5 \end{bmatrix} \xrightarrow[R1-R2, R1 \leftrightarrow R2]{-R1/18, R2/16} \begin{bmatrix} 1 & -5 & | & -5 \\ 0 & 1 & | & 1 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$\xrightarrow{R1+5R2} \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 1 \\ 0 & 0 & | & 0 \end{bmatrix}$, so $t = 0$, $s = 1$, and the only point of intersection is $(2, 2, 2)$ (by plugging $t = 0$ on the first line of $s = 1$ on the second one).

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1. Let $\vec{v} = 3\check{\mathbf{i}} - 2\check{\mathbf{j}} + \check{\mathbf{k}}$ and $\vec{w} = \check{\mathbf{i}} - 2\check{\mathbf{j}} + 3\check{\mathbf{k}}$.
   (a) Write the parametric equation of the following lines:
      i. the line parallel to $\vec{v} - 2\vec{w}$ and passing through $(1, 1, 1)$
      ii. the line parallel to $\vec{v} + 2\vec{w}$ and passing through $(5, 6, -1)$
   (b) Find, if any, the intersection points of the two lines above.

Answer. We calculate

$$\vec{v} - 2\vec{w} = \begin{bmatrix} 3 - 2 \times 1 \\ -2 - 2 \times (-2) \\ 1 - 2 \times 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix},$$

$$\vec{v} + 2\vec{w} = \begin{bmatrix} 3 + 2 \times 1 \\ -2 + 2 \times (-2) \\ 1 + 2 \times 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

The lines are then

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}, \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix} + s \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}.$$

In parametric form:

$$\begin{aligned} x &= 1 + t \\ y &= 1 + 2t \\ z &= 1 - 5t \end{aligned} \qquad \text{and} \qquad \begin{aligned} x &= 2 + 5s \\ y &= 3 - 6s \\ z &= -4 + 7s \end{aligned}$$

For the intersection, we need to solve the system

$$\begin{aligned} 1 + t &= 2 + 5s \\ 1 + 2t &= 3 - 6s \\ 1 - 5t &= -4 + 7s \end{aligned}$$

which we can write as

$$\begin{aligned} t - 5s &= 1 \\ 2t + 6s &= 2 \\ -5t + 7s &= -5 \end{aligned}$$

Doing row reduction,

$$\begin{bmatrix} 1 & -5 & | & 1 \\ 2 & 6 & | & 2 \\ -5 & 7 & | & -5 \end{bmatrix} \xrightarrow[R3+5R1]{R2-2R1} \begin{bmatrix} 1 & -5 & | & 1 \\ 0 & 16 & | & 0 \\ 0 & 32 & | & 0 \end{bmatrix} \xrightarrow[R2/16, R1+5R2]{R3-2R2} \begin{bmatrix} 1 & 0 & | & 1 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix},$$

so $t = 1$, $s = 0$, and the only point of intersection is $(2, 3, -4)$ (by plugging $t = 0$ on the first line of $s = 1$ on the second one).