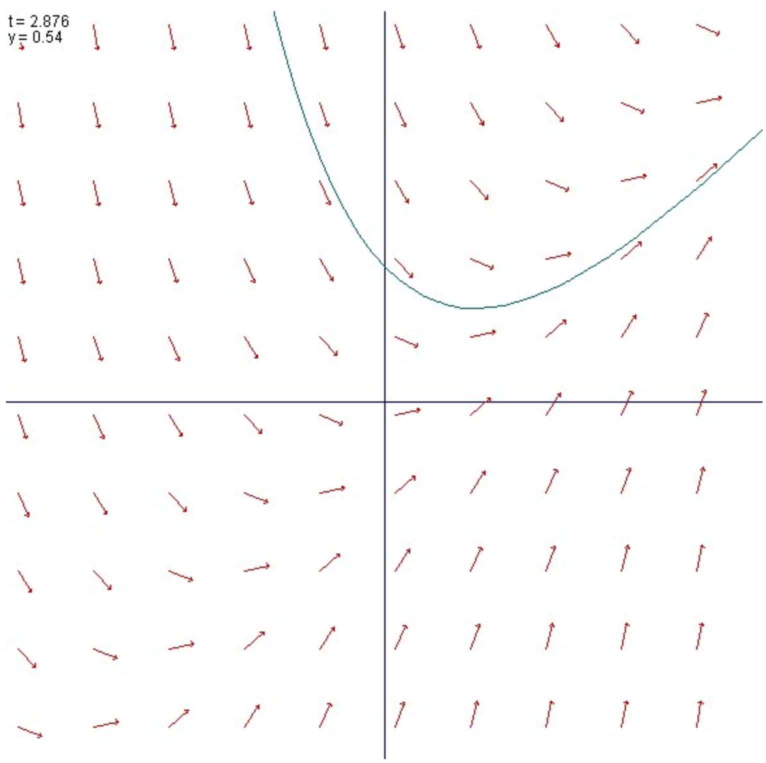
Math 217-001 201810

Practice Assignment # 2 – Answers

1. *(Download Picture # 1 for this question; in the graph, $-3 < x < 3$ and $-3 < y < 3$)* This is the graph of the direction field of a certain equation $y' = f(x, y)$.

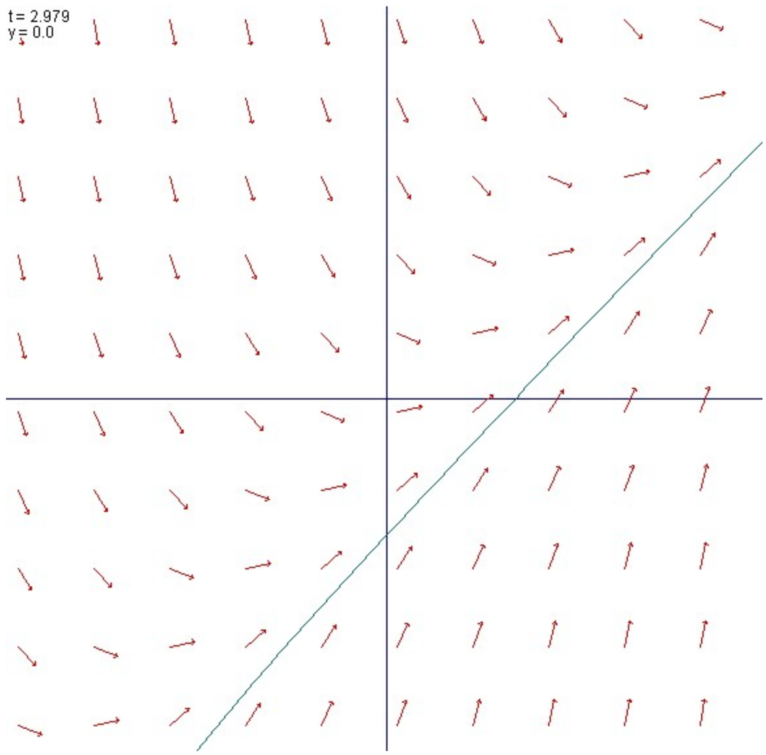
   (a) Draw an approximate solution y with $y(0) = 1$.

   Answer.



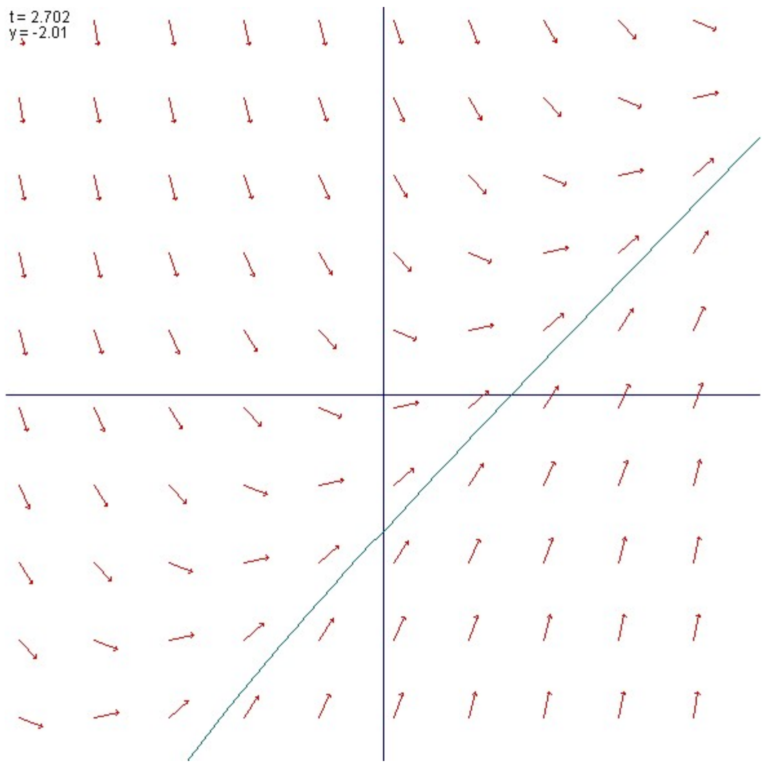
(b) Draw an approximate solution y with $y(1) = 0$.

Answer.



(c) Draw an approximate solution y with $y(0) = -1$.

Answer.



(d) Is it possible that $f(x, y) = y^2$?

Answer. No. Because if $f(x, y) = y^2$ then y' would be non-negative for all x: no arrow would point downwards.

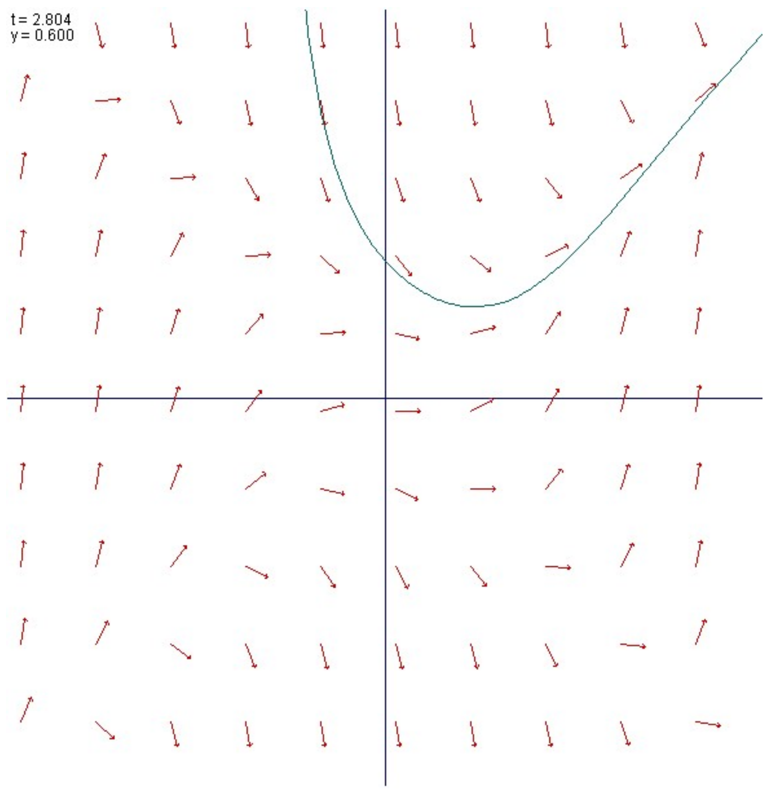
(e) Is it possible that $f(x, y) = x^2 + y^2$?

Answer. No, for the same reason: in the graph there are clearly many arrows showing a negative slope.

2. *(Download Picture # 2 for this question; in the graph,* $-3 < x < 3$ *and* $-3 < y < 3$*)* This is the graph of the direction field of a certain equation $y' = f(x,y)$.

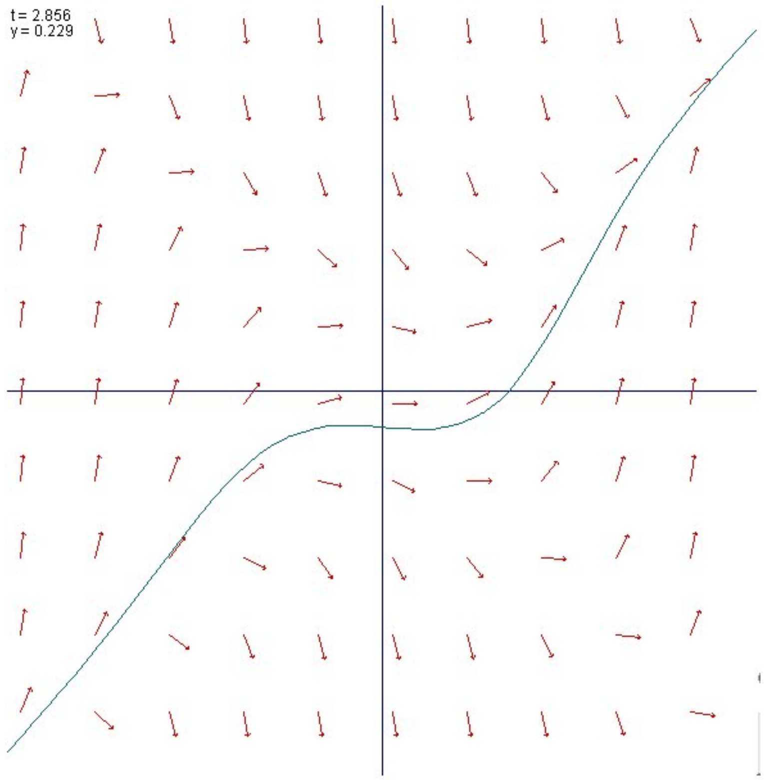
   (a) Draw an approximate solution y with $y(0) = 1$.

   Answer.



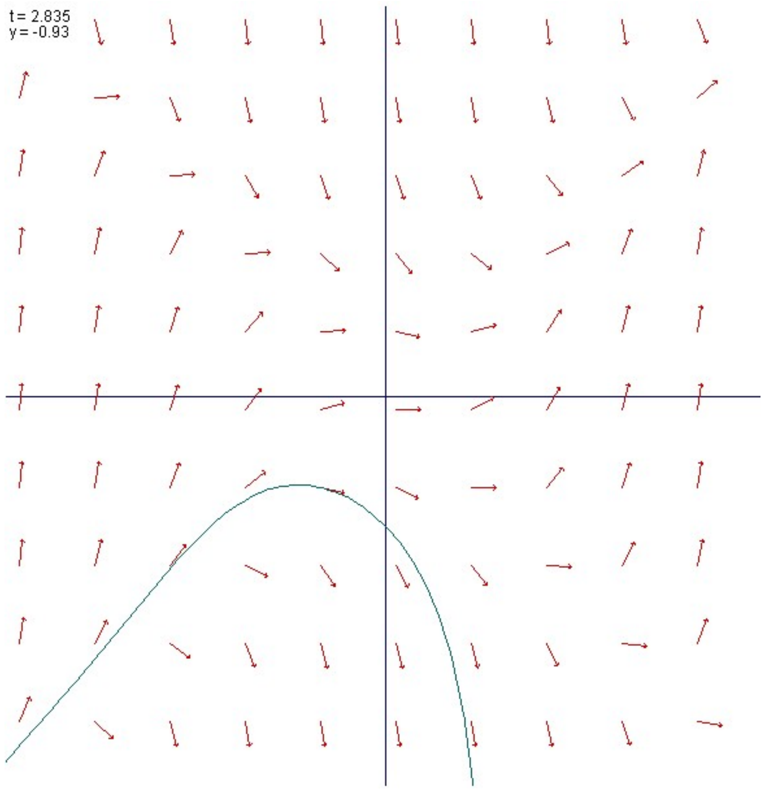
(b) Draw an approximate solution y with $y(1) = 0$.

Answer.



(c) Draw an approximate solution y with $y(0) = -1$.

Answer.



(d) Is it possible that $f(x, y) = y^2$?

Answer. No. Because if $f(x, y) = y^2$ then y' would be non-negative for all x: no arrow would point downwards.

(e) Is it possible that $f(x, y) = x^2 + y^2$?

Answer. No, for the same reason: in the graph there are clearly many arrows showing a negative slope.

3. For the equation $y' = y - y^3$:

(a) Find the critical points;

Answer. If $y - y^3 = 0$, we have $0 = y(1 - y^2)$, so either $y = 0$ or $y = 1$ or $y = -1$.

(b) decide if each critical point is semistable, attractor, or repeller;

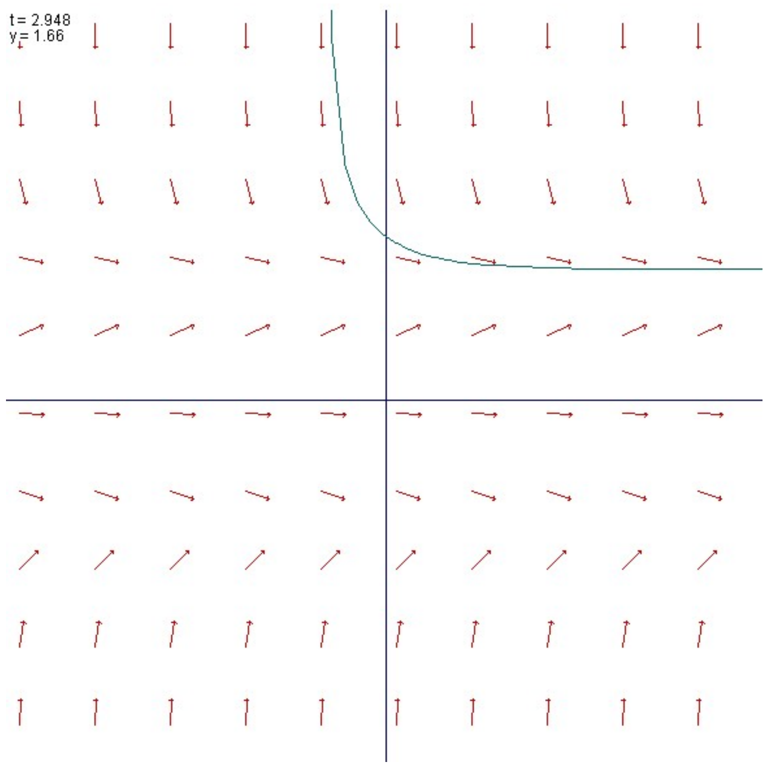
Answer. If $y > 1$, then $y > 0$ and $1 - y^2 < 0$, so $y - y^3 = y(1 - y^2) < 0$. If $0 < y < 1$, then $y(1 - y^2) > 0$. As $y - y^3 < 00$ above $y = 1$ and positive below, we deduce that $y = 1$ is an attractor.

When $-1 < y < 0$, $y < 0$ and $1 - y^2 > 0$, so $y - y^3 < 0$. So $y = 0$ is a repeller. When $y < -1$, $y - y^3 > 0$, so $y = -1$ is an attractor.

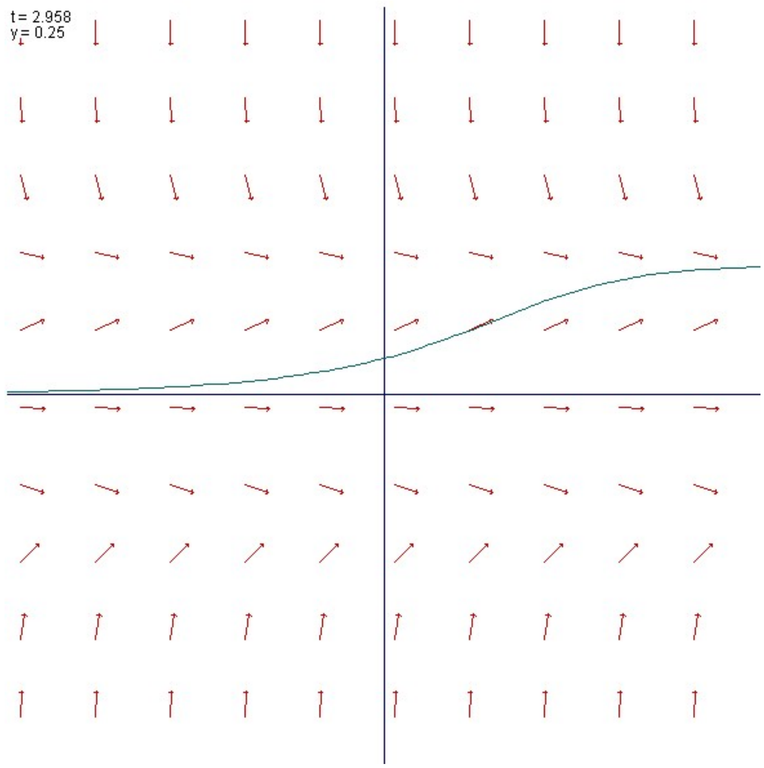
(c) draw possible solutions in each region as determined by the critical points.

Answer.

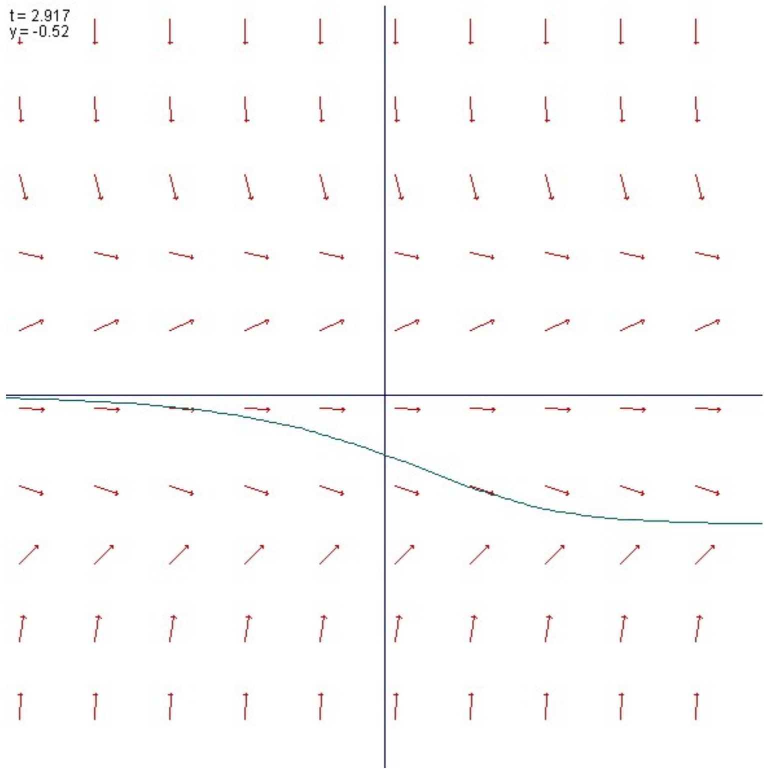
$1 < y$:



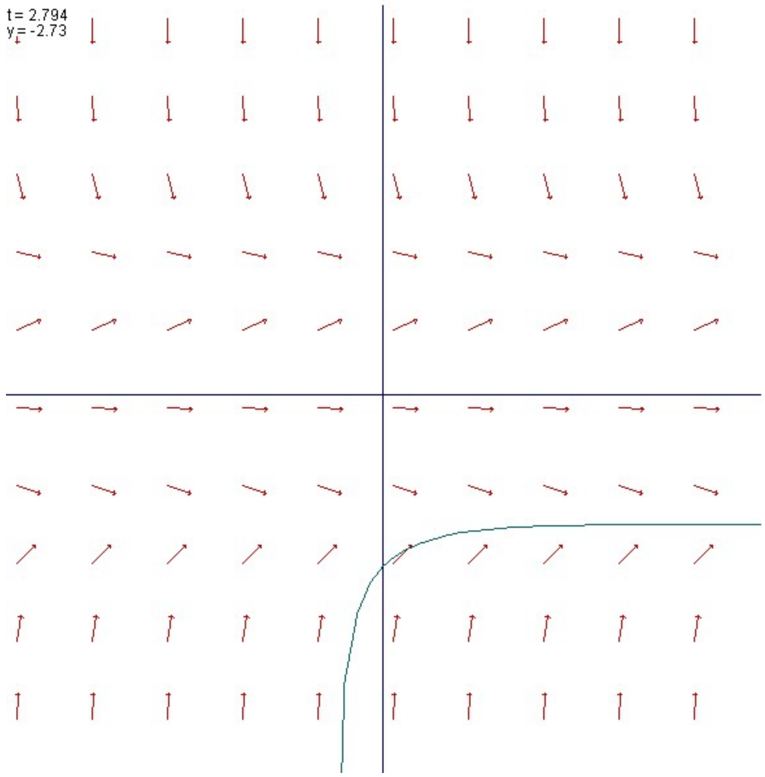
$0 < y < 1$:



$-1 < y < 0$:



$y < -1$:



4. Consider the initial value problem $y' = y/x$, $y(1) = -1/2$. Use Euler's method with $h = .1$ to approximate $y(2)$. Solve the separable equation to obtain y explicitly and compared with the value obtained.

Answer. For Euler's method, we take $h = .1$. We have the recursion $y_{n+1} = y_n + h\,f(x_n, y_n) = y_n + h\,y_n/x_n^2$. Now we calculate

n	$\mathbf{x_n}$	$\mathbf{y_n}$
0	1.0	-0.5000
1	1.1	-0.5500
2	1.2	-0.5955
3	1.3	-0.6368
4	1.4	-0.6745
5	1.5	-0.7089
6	1.6	-0.7404
7	1.7	-0.7693
8	1.8	-0.7959
9	1.9	-0.8205
10	2.0	-0.8432

So Euler method gives us an approximate value of $y(2) \simeq -0.8432$. If we solve the equation: as it is separable, $y'/y = 1/x^2$. Taking antiderivatives and the exponential, $y(x) = de^{-1/x}$. With the initial condition $y(1) = -1/2$, we deduce that

$$-\frac{1}{2} = y(1) = d\,e^{-1/1} = d\,e^{-1}.$$

So $e = -e/2$. That is $y(x) = -\dfrac{e^{1-1/x}}{2}$. In particular,

$$y(2) = -\frac{e^{1-1/2}}{2} = -\frac{e^{1/2}}{2} = -0.8244.$$