Math 217-001 201810
Practice Assignment # 5

1. (a) Show that $c_1 + c_2x^2$ is a two-parameter family of solutions of the equation $xy'' - y' = 0$, $x \in (-\infty, \infty)$.

   (b) Show that the initial value problem

   $$\begin{cases} xy'' - y' = 0, \\ y(0) = 0, \\ y'(0) = 1 \end{cases}$$

   has no solution.

   (c) Does the fact that the above IVP has no solution contradict the Existence and Uniqueness Theorem (4.1.1 in the textbook)?

2. (a) Show that $c_1 \cos x + c_2 \sin x$ is a two-parameter family of solutions of the equation $y'' + y = 0$.

   (b) Consider the BVP

   $$\begin{cases} y'' + y = 0, \\ y(0) = 0, \\ y(\pi) = 0 \end{cases}$$

   Show that the system has infinitely many solutions.

   (c) Does the fact that the above BVP has no solution contradict the Existence and Uniqueness Theorem (4.1.1 in the textbook)?

3. (a) Show that $y(x) = e^{3x}$ is a particular solution of $y'' - 2y' + 5y = 8e^{3x}$.

   (b) Show that $y(x) = 4x^3 + 2x$ is a particular solution of $y'' - 2y' + 5y = 20x^3 - 24x^2 + 34x - 4$.

   (c) Find a particular solution of $y'' - 2y' + 5y = e^{3x} - 20x^3 + 24x^2 - 34x + 4$.

4. Verify that the given functions form a fundamental set of solutions of the given equation on the indicated interval. Solve the IVP.

   (a) $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 1$; $\cos 2x$, $\sin 2x$, $(-\infty, \infty)$.

(b) $4y'' - 4y' + y = 0$, $y(0) = 4$, $y'(0) = 1$; $e^{x/2}$, $xe^{x/2}$, $(-\infty, \infty)$.

5. The function $y_1 = e^{-3x}$ is a solution of the associated homogeneous equation of the equation $y'' - 9y = 2$. Using reduction of order,

   (a) Find a second solution y_2 to the homogeneous equation;

   (b) Check that y_1 and y_2 form a fundamental set of solutions of the homogeneous equation.

   (c) Find a particular solution y_p to the equation $y'' - 9y = 2$.

   (d) Express the general solution of $y'' - 9y = 2$.

6. Decide if the functions are linearly independent.

   (a) $f_1(x) = x$, $f_2(x) = x^2$, $f_3(x) = x^3$.

   (b) $f_1(x) = \sin x$, $f_2(x) = \cos x$.

   (c) $f_1(x) = \sin^2 x$, $f_2(x) = \cos^2 x$.

   (d) $f_1(x) = \sinh x$, $f_2(x) = \cosh x$.

7. Solve the following initial value problems:

   (a) $y'' + 2y' + y = 0$, $y(1) = 2$, $y'(1) = -2$;

   (b) $y'' + 2y' = 0$, $y(1) = 2$, $y'(1) = -2$;

   (c) $y'' + 2y' + 2y = 0$, $y(1) = 2$, $y'(1) = -2$.

   (d) $x^2y'' + 3xy' + y = 0$, $y(1) = 2$, $y'(1) = -2$;

   (e) $x^2y'' + 3xy' = 0$, $y(1) = 2$, $y'(1) = -2$;

   (f) $x^2y'' + 3xy' + 2y = 0$, $y(1) = 2$, $y'(1) = -2$.

8. Show that $y_1(x) = 3e^{2x} - 1$ and $y_2(x) = e^{-x} + 2$ are solutions of $y\,y'' + 2y' - (y')^2 = 0$ but that neither $2y_1$ nor $y_1 + y_2$ is a solution. On the other hand, we proved in class that scalar multiplies and sums of solutions of linear DE are again solutions. Is there a contradiction ?